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$\hat{B}+\hat{C}=120 \rightarrow \hat{A}=60 \ , \ BC=20$

$\dfrac{\frac{\sqrt{3}}{2}\ \sin A}{20\ \ BC} = \dfrac{\sin B}{AC\ \ \frac{20\sqrt{6}}{3}} \rightarrow \sin B = \dfrac{\cancel{20}\sqrt{6}\times\sqrt{3}}{2\times 3\times \cancel{20}} = \dfrac{\sqrt{18}}{6} = \dfrac{\cancel{3}\sqrt{2}}{\cancel{6}} = \dfrac{\sqrt{2}}{2} \rightarrow \boxed{B=45^\circ}$

$\xrightarrow{B+C=120} C=75^\circ$

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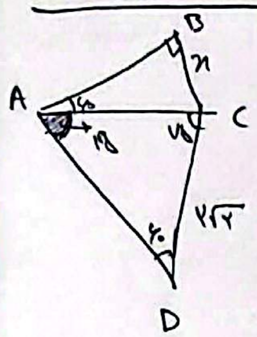

$\dfrac{\sin 45}{2\sqrt{2}} = \dfrac{\sin 60}{AC} \rightarrow \dfrac{\sqrt{2}}{2\times 2\sqrt{2}} = \dfrac{\sqrt{3}}{2AC} \rightarrow \boxed{AC = \dfrac{2\sqrt{3}}{\sqrt{2}} = \sqrt{6}}$

$\sin 60 = \dfrac{x}{AC} \rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{x}{\sqrt{6}} \rightarrow x = \dfrac{\sqrt{18}}{2} = \dfrac{3\sqrt{2}}{2}$

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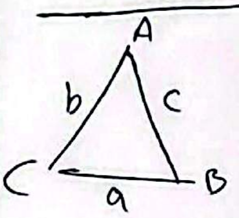

$(b^2-2)\dfrac{\sin C}{\sin B} = c \rightarrow (b^2-2)\sin C = c\cdot\sin B \rightarrow \dfrac{\sin B}{b^2-2} = \dfrac{\sin C}{c}$

$\xrightarrow{\frac{\sin B}{b}=\frac{\sin C}{c}} b^2-2=b \rightarrow b^2-b-2=0 \rightarrow (b-2)(b+1)=0 \rightarrow b=2$

$\rightarrow b=-1 \times$

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$AC^2 = AB^2+BC^2-2\times AB\times BC\times\cos B \rightarrow 169 = \underbrace{400+441}_{841} - \underbrace{20\times 21\times 2\times\cos B}_{-840\cos B}$

$\rightarrow 840\cos B = 672 \rightarrow \cos B = \dfrac{672}{840} = \boxed{\dfrac{4}{5}} \xrightarrow{\sin^2+\cos^2=1} \sin^2 B + \dfrac{16}{25} = 1 \rightarrow \sin^2 B = \dfrac{9}{25}$

$\rightarrow \sin B = \dfrac{3}{5}$

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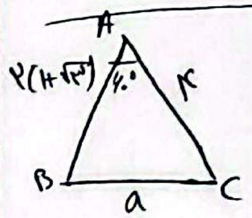

الف) $a = BC = \sqrt{16+16+8\sqrt{3}-16(1+\sqrt{3})\times\frac{1}{2}} = \sqrt{24} = 2\sqrt{6}$

ب) $\dfrac{AC}{\sin B} = \dfrac{BC}{\sin 60} \rightarrow \dfrac{4}{\sin B} = \dfrac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} \rightarrow \sin B = \dfrac{\cancel{4}\sqrt{3}}{\cancel{4}\sqrt{6}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \rightarrow B=45$

$C = 180-(60+45) = 75^\circ$

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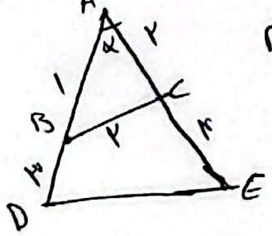

$BC = 2 = \sqrt{4+1-4\cos\alpha} \rightarrow 5-4\cos\alpha = 4 \rightarrow 4\cos\alpha = 1 \rightarrow \boxed{\cos\alpha = \dfrac{1}{4}}$

$DE = \sqrt{16+36-\dfrac{\cancel{48}\times 1}{\cancel{4}}} = \sqrt{40} = 2\sqrt{10}$

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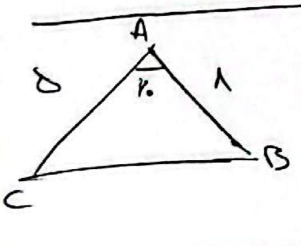

الف) $BC = \sqrt{64+25-\cancel{80}\times\dfrac{1}{\cancel{2}}} = \sqrt{49} = 7$

ب) $S = \dfrac{1}{2}AC\cdot AB\cdot\sin 60 = \dfrac{\cancel{8}\times 5\times\sqrt{3}}{\cancel{2\times 2}} = 10\sqrt{3}$

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$$\frac{a^3+b^3-c^3}{a+b-c} = a^2 \to \cancel{a^3}+b^3-c^3 = \cancel{a^3} + a^2 b - a^2 c \to \cancel{(b-c)}(b^2+c^2+bc) = a^2\cancel{(b-c)}$$

$$\to \begin{cases} b^2+c^2+bc = a^2 \\ a^2 = b^2+c^2-2bc\cos A \end{cases}$$ از طرفی می‌دانیم

$$\Rightarrow \cancel{b^2}+\cancel{c^2}+bc = \cancel{b^2}+\cancel{c^2}-2bc\cos A \to -2\cancel{bc}\cos A = \cancel{bc}$$

$$\to -2\cos A = 1 \to \cos A = -\frac{1}{2}$$

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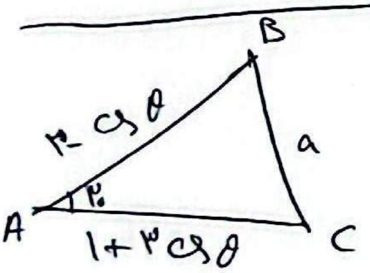

$$S = 2 = \cancel{\tfrac{1}{2} \times \sin 30} \times \underbrace{(3-\cos\theta)(1+3\cos\theta)}_{8}$$

$$\to -3\cos^2\theta + 8\cos\theta + 3 = 8 \to \underbrace{-3}_{a}\cos^2\theta + \underbrace{8}_{b}\cos\theta \underbrace{-5}_{c} = 0$$

$$\to a+b+c=0 \to \begin{cases} \cos\theta = 1 \checkmark \to \boxed{AB = 3},\ \boxed{AC = 4} \\ \cos\theta = \frac{5}{3} \times \end{cases}$$

$$a^2 = AB^2 + AC^2 - 2AB.AC.\cos 30 \to a^2 = 9+16-24\times\frac{\sqrt{3}}{2} = 25-12\sqrt{3}$$