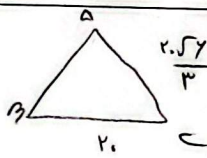


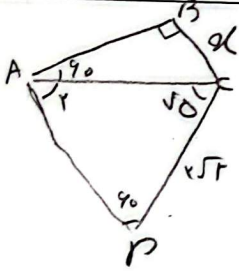
$BC = 2$, $AC = \frac{2\sqrt{3}}{3}$
 $\hat{A} + \hat{C} = 120^\circ$



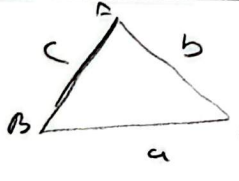
$B + C + A = 180^\circ \rightarrow 120^\circ + A = 180^\circ \rightarrow A = 60^\circ$
 $\rightarrow \sin B = \frac{\sqrt{3}}{2} \rightarrow B = 60^\circ$
 $B + C = 120^\circ \rightarrow 60^\circ + C = 120^\circ \rightarrow C = 60^\circ$

$\frac{BC}{\sin A} = \frac{AC}{\sin B} \rightarrow \frac{2}{\sin 90^\circ} = \frac{\frac{2\sqrt{3}}{3}}{\sin B} \rightarrow \sin B = \frac{\sqrt{3}}{2}$
 $B = 60^\circ$

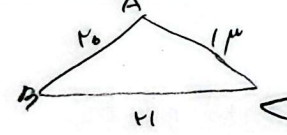
$B + C = 120^\circ \rightarrow 60^\circ + C = 120^\circ \rightarrow C = 60^\circ$



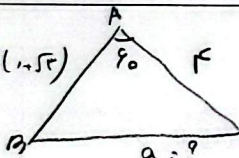
$AD + DC + AC = 120^\circ \rightarrow AD = 60^\circ$
 $\frac{DC}{\sin AD} = \frac{AC}{\sin D} \rightarrow \frac{DC}{\sin 60^\circ} = \frac{\frac{2\sqrt{3}}{3}}{\sin 90^\circ} \rightarrow DC = \frac{2\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{2} = 1$
 $\rightarrow \sin 40^\circ = \frac{BC}{AC} = \frac{DC}{\frac{2\sqrt{3}}{3}} \rightarrow \frac{2}{\frac{2\sqrt{3}}{3}} = \frac{1}{\frac{2\sqrt{3}}{3} \sin 40^\circ} \rightarrow \sin 40^\circ = \frac{\sqrt{3}}{2}$



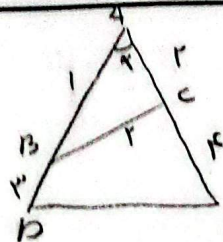
$(b^2 - r^2) \frac{\sin C}{\sin B} = C$
 $\frac{b}{\sin B} = \frac{C}{\sin C}$
 $\frac{1}{\sin C} \Rightarrow \frac{b^2 - r^2}{\sin B} = \frac{C}{\sin C} = \frac{b}{\sin B} \rightarrow b^2 - r^2 = b$
 $b^2 - b - r^2 = 0 \rightarrow (b - r)(b + r) = 0 \rightarrow b = r$



$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos B$
 $149 = 800 + 881 - 180 \cos B$
 $\rightarrow \cos B = \frac{1}{2} \rightarrow \sin^2 B + \cos^2 B = 1 \rightarrow \sin B = \frac{\sqrt{3}}{2}$



$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 4 + 1 + 1 - 1 = 3$
 $a = \sqrt{3}$
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{\sqrt{3}}{\sin 90^\circ} = \frac{2}{\sin B} \rightarrow \sin B = \frac{\sqrt{3}}{2}$
 $B = 60^\circ$
 $B + C = 120^\circ \rightarrow 60^\circ + C = 120^\circ \rightarrow C = 60^\circ$

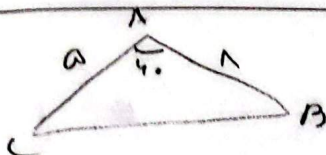


$$AB^2 \rightarrow BC^2 = AC^2 + AB^2 - 2 \times AC \times AB \cos \alpha$$

$$\rightarrow 4 = 1 + 1 - 2 \times 1 \times 1 \cos \alpha \rightarrow \cos \alpha = \frac{1}{2}$$

$$ADE \rightarrow DE^2 = AD^2 + AE^2 - 2 \times AD \times AE \cos \alpha$$

$$\rightarrow DE^2 = 1 + 1 - 2 \times 1 \times \frac{1}{2} \times \frac{1}{2} \rightarrow DE = \sqrt{1} = 1$$



$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \cos \alpha$$

$$\rightarrow 49 = 16 + 25 - 2 \times 4 \times 5 \cos \alpha$$

$$\rightarrow \cos \alpha = \frac{1}{4}$$

$$S = \frac{1}{2} \times AC \times AB \times \sin \alpha = \frac{1}{2} \times 5 \times 4 \times \frac{\sqrt{15}}{4} = 5\sqrt{15}$$

$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)}$$

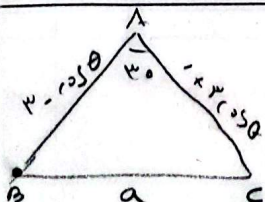
$$\rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$\rightarrow \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A}{bc(1 + \cos A)} = \frac{2bc(1 + \cos A)}{bc(1 + \cos A)} = 2$$

$$\frac{a^3 + b^3 - c^3}{a+b-c} \geq a^2 \rightarrow a^3 + b^3 - c^3 \geq a^2(a+b-c)$$

$$\rightarrow (b-c)(b^2 + c^2 + bc) \geq a^2(b-c) \xrightarrow{a \neq c} a^2 \geq b^2 + c^2 + bc$$

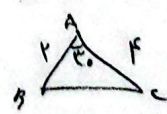
$$\xrightarrow{\text{فرض کنیم}} a^2 \geq b^2 + c^2 - 2bc \cos A \rightarrow \cos A \geq \frac{1}{2} \rightarrow A \leq 60^\circ$$



$$S = \frac{1}{2} \times AB \times AC \times \sin \theta \rightarrow \frac{1}{2} \times (1 - \cos \theta)(1 + \cos \theta) \geq \frac{1}{2}$$

$$\rightarrow 1 - \cos^2 \theta \geq 1 \rightarrow -\cos^2 \theta \geq 0 \rightarrow \cos \theta = 0 \rightarrow \theta = 90^\circ$$

$$\cos \theta \geq \frac{1}{2} \rightarrow \cos \theta \geq \frac{1}{2} \rightarrow \cos \theta \geq \frac{1}{2}$$



$$\rightarrow a^2 \geq AB^2 + AC^2 - 2 \times AB \times AC \times \cos \theta \rightarrow a^2 \geq 1 + 1 - 2 \times 1 \times 1 \times \frac{1}{2} = 1 \rightarrow a \geq 1$$