

$$\hat{B} + \hat{C} = 110^\circ \rightarrow \hat{A} = 70^\circ$$

$$BC = 20$$

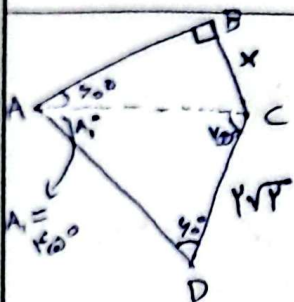
$$AC = \frac{20\sqrt{5}}{2}$$

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \Rightarrow \frac{20}{\sin 70^\circ} = \frac{20\sqrt{5}}{\sin B}$$

$$\sin B = \frac{\sqrt{5}}{2}$$

$$\hat{C} = 70^\circ$$

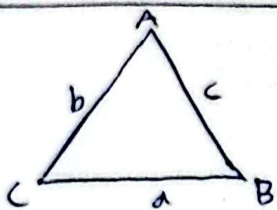
$$\hat{B} = 40^\circ$$



$$\frac{CD}{\sin \hat{A}_1} = \frac{AC}{\sin \hat{B}} \rightarrow \frac{20\sqrt{5}}{\sin 70^\circ} = \frac{AC}{\sin 40^\circ}$$

$$AC = 20\sqrt{5}$$

$$AC \times \sin 40^\circ = BC \rightarrow BC = 20$$

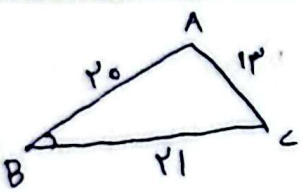


$$(b^2 - r^2) \frac{\sin \hat{C}}{\sin \hat{B}} = c$$

$$\frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b}$$

$$(b^2 - r^2) \frac{c}{b} = c$$

$$b^2 - b - r^2 = 0 \rightarrow b^2 - r^2 \leq b$$



$$\sin \hat{B} \leq 1$$

$$AC = \sqrt{AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos B}$$

$$20\sqrt{5} = \sqrt{20^2 + 20^2 - 2 \cdot 20 \cdot 20 \cdot \cos B}$$

$$\sin B \leq 0.5$$

$$\cos B = 0.1$$

$$20\sqrt{5} = 20 \cos B$$

$$b = r$$

$$c = r + r\sqrt{5}$$

$$\hat{A} = 40^\circ$$

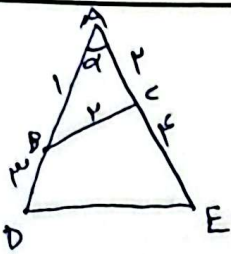
$$r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos A = a^2$$

$$1 + (1 + \sqrt{5}) - (1 + \sqrt{5}) = 2r^2 = a^2$$

$$\frac{r}{\sin B} = \frac{2r\sqrt{5}}{\sin A}$$

$$\sin B = \frac{\sqrt{5}}{2} \rightarrow \hat{B} = 70^\circ$$

$$\hat{C} = 180 - (A + B) = 70^\circ = \hat{C}$$



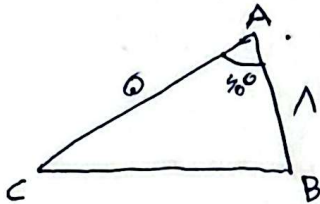
$$AC^2 + AB^2 - 2 \cdot AC \cdot AB \cdot \cos \alpha = BC^2$$

$$\frac{P^2 + P^2 - 2 \cdot P \cdot P \cdot 1 \cdot \cos \alpha = BC^2}{\omega}$$

$$\hookrightarrow P \cos \alpha \leq 1 \rightarrow \cos \alpha \leq \frac{1}{P}$$

$$DE^2 = AD^2 + AE^2 - 2 \cdot AD \cdot AE \cdot \cos \alpha$$

$$\frac{P^2 + P^2 - 2 \cdot P \cdot P \cdot \frac{1}{P} = BC^2}{\frac{1}{P}} \rightarrow DE = P \sqrt{10}$$



$$BC^2 = AC^2 + AB^2 - 2 \cdot AB \cdot AC \cdot \cos 90^\circ$$

$$\frac{P^2 + P^2 - 2 \cdot P \cdot P \cdot 0 = BC^2}{\frac{1}{P}} \rightarrow BC = P$$

$$S_{ABC} = \frac{AC \cdot AB \cdot \sin 90^\circ}{2} \rightarrow \frac{P \times P \times \frac{\sqrt{2}}{2}}{2} = \frac{10\sqrt{2}}{2} = S_{ABC}$$

$$\frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} \rightarrow \frac{b^2 + 2bc + c^2 - (b^2 + c^2 - 2bc \cos \hat{A})}{bc(\cos \hat{A} + 1)} \rightarrow$$

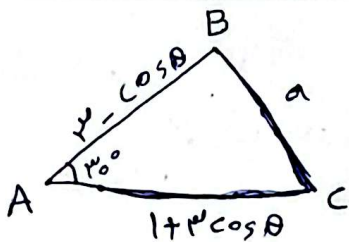
$$\rightarrow \frac{2bc + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(1 + \cos \hat{A})}{bc(\cos \hat{A} + 1)} = 2$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a \rightarrow a^2 + b^2 - c^2 = a^2 + a^2b - a^2c$$

$$(b-c)(b^2 + bc + c^2)$$

$$\Rightarrow (b-c)(b^2 + bc + c^2) = a^2(b-c) \rightarrow b^2 + bc + c^2 = a^2$$

$$\Rightarrow b^2 + bc + c^2 = b^2 + c^2 - 2bc \cos \hat{A} \rightarrow 1 = -2 \cos \hat{A} \rightarrow \cos \hat{A} = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$$



$$S_{ABC} = \frac{AB \cdot AC \cdot \sin 120^\circ}{2} = \frac{1 \cdot 1 \cdot \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{4}$$

$$\rightarrow \frac{(1 - \cos \theta)(1 + \cos \theta) \times \frac{1}{2}}{2} = \frac{\sqrt{3}}{4} \rightarrow -\cos \theta + \cos \theta + 1 = \sqrt{3}$$

$$\rightarrow 1 - \cos \theta = \sqrt{3} - 1 \rightarrow \cos \theta = \frac{2 - \sqrt{3}}{2} \rightarrow \theta = 15^\circ$$

$$a^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos \hat{A} \rightarrow 1^2 + 1^2 - 2 \cdot 1 \cdot 1 \cdot \cos 120^\circ = 3 \rightarrow a = \sqrt{3}$$