نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ..۱۳.. کلاس ....

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$\hat{B} + \hat{C} = 120^\circ \to \hat{A} = 60^\circ$

$BC = 20$

$AC = \frac{20\sqrt{6}}{3}$

$\frac{BC}{\sin A} = \frac{AC}{\sin B} \Rightarrow \frac{20}{\frac{\sqrt{3}}{2}} = \frac{\frac{20\sqrt{6}}{3}}{\sin B}$

$\sin B = \frac{\sqrt{2}}{2}$

$\hat{B} = 45^\circ \to \hat{C} = 75^\circ$

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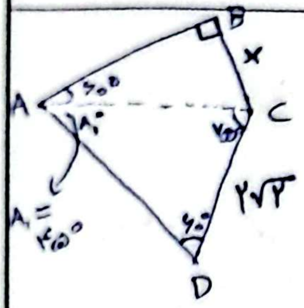

$\frac{CD}{\sin \hat{A}_1} = \frac{AC}{\sin \hat{D}} \to \frac{2\sqrt{2}}{\sin 45^\circ} = \frac{AC}{\sin 60^\circ}$, $\sin 45^\circ = \frac{\sqrt{2}}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$AC = 2\sqrt{3}$

$AC \times \sin 60^\circ = BC \to BC = x = 3$

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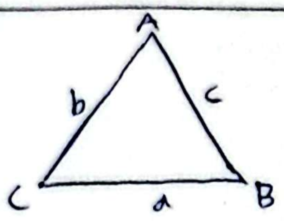

$(b^2 - 2)\frac{\sin \hat{C}}{\sin \hat{B}} = c$

$\frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b} \to (b^2 - 2)\frac{\cancel{c}}{b} = \cancel{c}$

$b^2 - 2 = b \to b^2 - b - 2 = 0$

$b = 2$

$\cancel{b = -1}$ غ ق ق

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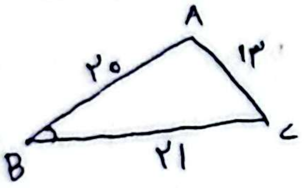

$\sin \hat{B} = ?$

$AC = \sqrt{AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos B}$

$13^2 = 20^2 + 21^2 - 2 \cdot 20 \cdot 21 \cdot \cos B$

$169 = 841 - 840 \cos B \to 672 = 840 \cos B$

$\cos B = 0.8$

$\sin B = 0.6$

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$b = 4$

$c = 2 + 2\sqrt{3}$

$\hat{A} = 60^\circ$

الف) $a = ?$

$b^2 + c^2 - 2bc \cos \hat{A} = a^2$, $\cos 60^\circ = \frac{1}{2}$

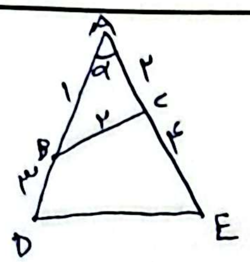
$16 + (4 + 8\sqrt{3} + 12) - (8 + 8\sqrt{3}) = 24 = a^2$

$a = 2\sqrt{6}$

ب) $\hat{B} = ?$ $\hat{C} = ?$

$\frac{4}{\sin \hat{B}} = \frac{2\sqrt{6}}{\sin \hat{A}}$, $\sin \hat{A} = \frac{\sqrt{3}}{2}$ $\to \sin \hat{B} = \frac{\sqrt{2}}{2} \to \hat{B} = 45^\circ$

$\hat{C} = 180 - (A + B) = 75^\circ = \hat{C}$



$$AC^2 + AB^2 - 2.AC.AB.\cos\alpha = BC^2$$

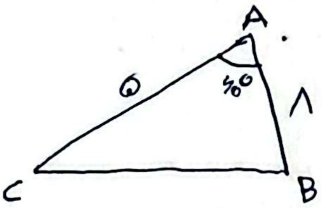
$$\cancel{2^2 + 1^2}_{5} - 2.2.1.\cos\alpha = \cancel{4}$$

$$\rightarrow 4\cos\alpha = 1 \rightarrow \cos\alpha = \frac{1}{4}$$ ۶

$$DE^2 = AD^2 + AE^2 - 2.AD.AE.\cos\alpha$$

$$16 + 36 - 2.4.6.\frac{1}{4} = 40 \rightarrow DE = 2\sqrt{10}$$

الف)



$$BC^2 = AC^2 + AB^2 - 2.AB.AC.\cos 60°$$

$$\cancel{25 + 64}_{89} - \cancel{2.8.5}_{40}.\frac{1}{2} = 49 \rightarrow BC = 7$$ ۷

$$S_{ABC} = \frac{AC.AB.\sin\hat{A}}{2} \rightarrow \frac{5 \times 8 \times \frac{\sqrt{3}}{2}}{2} = 10\sqrt{3} = S_{ABC}$$

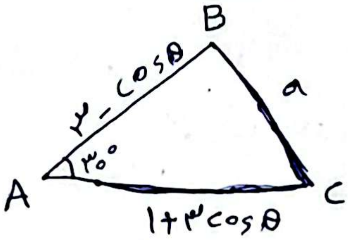
$$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} \rightarrow \frac{\cancel{b^2} + 2bc + \cancel{c^2} - (\cancel{b^2} + \cancel{c^2} - 2bc.\cos\hat{A})}{bc(\cos\hat{A}+1)} \rightarrow$$

$$\rightarrow \frac{2bc + 2bc.\cos\hat{A}}{bc(\cos\hat{A}+1)} = \frac{2\cancel{bc}\cancel{(1+\cos\hat{A})}}{\cancel{bc}\cancel{(\cos\hat{A}+1)}} = 2$$ ۸

$$\frac{a^3 + b^3 - c^3}{a+b-c} = a^2 \rightarrow \cancel{a^3} + \underbrace{b^3 - c^3}_{(b-c)(b^2+bc+c^2)} = \cancel{a^3} + a^2b - a^2c$$

$$\Longrightarrow \cancel{(b-c)}(b^2+bc+c^2) = \underbrace{a^2}_{b^2+c^2-2bc.\cos\hat{A}}\cancel{(b-c)}$$

$$\Longrightarrow \cancel{b^2} + \cancel{b}\cancel{c} + \cancel{c^2} = \cancel{b^2} + \cancel{c^2} - 2\cancel{bc}.\cos\hat{A} \rightarrow 1 = -2\cos\hat{A} \rightarrow \cos\hat{A} = -\frac{1}{2}$$



$$S_{ABC} = \frac{AB.AC.\sin 30°}{2} = 2$$

$$\rightarrow \frac{(3-\cos\theta)(1+3\cos\theta) \times \frac{1}{2}}{2} = 2 \rightarrow -3\cos^2\theta + 8\cos\theta + 3 = 8$$ ۱۰

$$\rightarrow 3\cos^2\theta - 8\cos\theta + 5 = 0 \quad \begin{cases} \cos\theta = \frac{5}{3} & \text{غ‌ق‌ق} \\ \cos\theta = \frac{3}{3} = 1 \rightarrow & AB = 2,\ AC = 4 \end{cases}$$

$$a^2 = AB^2 + AC^2 - 2.AB.AC.\cos\hat{A} \rightarrow \cancel{4} + \cancel{16} - 2.2.4.\frac{\sqrt{3}}{\cancel{2}} = 20 - 8\sqrt{3} = a^2$$