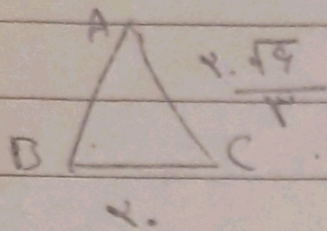


دائرة عمودية - كافي ١٣

مسألة ٢٠

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$$\hat{B} + \hat{C} = 180^\circ - \hat{A} = 90^\circ \rightarrow \frac{BC}{\sin A} = \frac{AC}{\sin B}$$

$$\rightarrow \frac{2}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{\sin B} \rightarrow \sin B = \frac{\sqrt{3}}{2} \rightarrow \hat{B} = 60^\circ$$

$$e = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$\hat{A}_1 = 180^\circ - (90^\circ + 30^\circ) = 60^\circ \quad \frac{DC}{\sin 60^\circ} = \frac{AC}{\sin 90^\circ} \rightarrow \frac{\sqrt{3}}{2} = \frac{AC}{1}$$

$$\rightarrow AC = \sqrt{3} \rightarrow x = \sin 90^\circ \times \sqrt{3}$$

$$\frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3$$

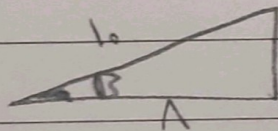
$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b} \rightarrow (b^2 - c^2) \times \frac{c}{b} = c \rightarrow$$

$$b^2 - b - c = 0 \rightarrow (b - c)(b + 1) = 0 \rightarrow b = -1 \text{ (invalid)}$$

$$b = 2\sqrt{3}$$

$$AC^2 = AD^2 + DC^2 - (2 \times AD \times DC \times \cos B) \rightarrow 1^2 = 2^2 + 3 - (2 \times 2 \times \sqrt{3} \times \cos B)$$

$$\rightarrow \cos B = \frac{1}{2}$$



$$1^2 - 2^2 = -3 \rightarrow 3 = 2 \times 2 \times \sqrt{3} \times \cos B \rightarrow \sin B = \frac{1}{2}$$

$$a^2 = b^2 + c^2 - 2ab \cos A = 1^2 + (2 + 2\sqrt{3})^2 - (2 \times 1 \times (2 + 2\sqrt{3}) \times \frac{1}{2}) = 14$$

$$\rightarrow a = \sqrt{14} = 2\sqrt{3.5}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{2\sqrt{3.5}}{\frac{\sqrt{3}}{2}} = \frac{2}{\sin B} = \frac{2 + 2\sqrt{3}}{\sin C}$$

$$\sin B = \frac{\sqrt{3}}{2} \rightarrow \hat{B} = 60^\circ \quad e = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$BC^2 = 1^2 + 4^2 - (1 \times 4 \times \cos \alpha) = 17 \rightarrow \cos \alpha = \frac{1}{4}$$

$$\rightarrow DE^2 = 1^2 + 4^2 - (1 \times 4 \times \cos \alpha) = 17 \rightarrow DE = \sqrt{17} = 4\sqrt{1}$$

$$BC^2 = 2^2 + 1^2 - (2 \times 1 \times \cos 90^\circ) = 5 \rightarrow BC = \sqrt{5}$$

$$S = \frac{1}{2} \times 2 \times 1 \times \sin 90^\circ = \frac{2 \times 1 \times 1}{2} = 1$$

$$\frac{b^2 + c^2 + 2bc - a^2}{bc \cos A + bc} = \frac{b^2 + c^2 + 2bc - a^2 - b^2 + 2bc \cos A}{bc (\cos A + 1)}$$

$$\frac{2bc(1 + \cos A)}{bc(1 + \cos A)} = 2$$

$$a^2 + b^2 - c^2 = a^2 + a^2b - a^2c \rightarrow (b-c)(b^2 + c^2 + bc) = a^2(b-c)$$

$$\rightarrow a^2 = b^2 + c^2 + bc \quad bc = -2bc \cos A \rightarrow \cos A = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$$

$$\frac{1}{2} \times (3 - \cos \alpha) (1 + 3 \cos \alpha) \times \sin 120^\circ = 2 \rightarrow \Delta = (3 - \cos \alpha) (1 + 3 \cos \alpha) = 4$$

$$\Delta = 3 + 9 \cos \alpha - \cos \alpha - 3 \cos^2 \alpha \rightarrow 3 \cos^2 \alpha - \Delta \cos \alpha + 2 = 0 \rightarrow \begin{cases} \cos = 1 \checkmark \\ \cos = \frac{2}{3} \times \end{cases}$$

$$\rightarrow a^2 = 2^2 + 3^2 - (2 \times 3 \times \cos 120^\circ) = 13 \rightarrow a = \sqrt{13}$$

بهذا