پویا منتظریان

به نام خدا

DATE / / SUBJECT:

① $B+C=120^\circ \rightarrow A=60^\circ \rightarrow \dfrac{BC}{\sin A}=\dfrac{AC}{\sin B} \rightarrow \dfrac{20}{\frac{\sqrt{3}}{2}}=\dfrac{\frac{20\sqrt{6}}{3}}{\sin B} \rightarrow \dfrac{2}{\sqrt{3}}=\dfrac{\sqrt{6}}{3\sin B} \rightarrow \sin B=\dfrac{\sqrt{2}}{2} \rightarrow$

$\boxed{B=45^\circ} \rightarrow \boxed{C=75^\circ}$

$B=135^\circ$ غ ق ق

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② $\dfrac{CD}{\sin A}=\dfrac{AC}{\sin 60} \rightarrow \dfrac{2\sqrt{2}}{\frac{\sqrt{2}}{2}}=\dfrac{AC}{\frac{\sqrt{3}}{2}} \rightarrow AC=2\sqrt{3} \rightarrow$

$BC=AC\sin 60^\circ=2\sqrt{3}\times\dfrac{\sqrt{3}}{2}=\boxed{3=x}$

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③ $(b^2-2)\dfrac{\sin\hat{C}}{\sin\hat{B}}=c \xrightarrow{\div \sin\hat{C}} \dfrac{b^2-2}{\sin\hat{B}}=\dfrac{c}{\sin\hat{C}}=\dfrac{b}{\sin\hat{B}} \rightarrow b^2-2=b \rightarrow$

$b^2-b-2=0 \rightarrow \boxed{b=2}$

$b=-1$ غ ق ق

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④ $(21)^2+(20)^2-2(20)(21)\times\cos\hat{B}=(13)^2 \rightarrow 841-840\cos\hat{B}=169 \rightarrow$

$840\cos\hat{B}=672 \rightarrow \cos\hat{B}=\dfrac{672}{840}=\dfrac{168}{210}=\dfrac{56}{70}=\dfrac{8}{10} \rightarrow \sin\hat{B}=\dfrac{6}{10}$

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⑤ الف) $a^2=b^2+c^2-2bc\cos\hat{A} \rightarrow a^2=16+4(4+2\sqrt{3})-16(1+\sqrt{3})\times\dfrac{1}{2}=$

$32+8\sqrt{3}-8-8\sqrt{3}=24 \rightarrow a=\sqrt{24}=2\sqrt{6}$

ب) $\dfrac{4}{\sin\hat{B}}=\dfrac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} \rightarrow \sin B=\dfrac{\sqrt{2}}{2} \rightarrow \boxed{B=45^\circ} \rightarrow \hat{C}=180-(45+60)=\boxed{75^\circ}$

پویا منلیان

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۶) $\overset{\triangle}{ABC}$ در: $AC^2+AB^2-2(AB)(AC)\cos\alpha = BC^2 \to 4=4+1-4\cos\alpha \to \cos\alpha=\frac{1}{4}$

$\to DE^2=AD^2+AE^2-2(AD)(AE)\cos\alpha \to DE^2=16+36-48\times\frac{1}{4}=40 \to \boxed{DE=2\sqrt{10}}$

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۷) الف) $BC^2=64+25-80\times\frac{1}{2}=49 \to \boxed{BC=7}$

ب) $S_{\overset{\triangle}{ABC}}=\frac{1}{2}\times AB\times AC\times \sin 60^\circ=\frac{1}{2}\times 8\times 5\times\frac{\sqrt{3}}{2}=\boxed{10\sqrt{3}}$

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۸) $\frac{b^2+c^2+2bc-a^2}{bc(\cos\hat{A}+1)}=\frac{b^2+c^2+2bc-(b^2+c^2-2bc\cos\hat{A})}{bc(\cos\hat{A}+1)}=\frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)}=2$

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۹) $a^3+b^3-c^3=a^3+a^2(b-c) \to b^3-c^3=a^2(b-c) \to (b-c)(b^2+bc+c^2)=a^2(b-c)$

$\to \left.\begin{array}{l} a^2=b^2+c^2+bc \\ a^2=b^2+c^2-2bc\cos\hat{A} \end{array}\right\} \to -2bc\cos\hat{A}=bc \to \boxed{\cos\hat{A}=-\frac{1}{2}}$

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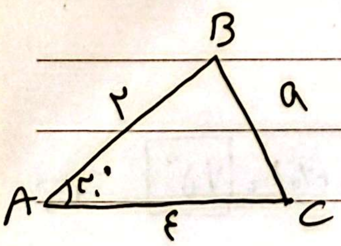
۱۰) $\frac{1}{2}\times(3-\cos\theta)(1+3\cos\theta)\times\frac{1}{2}=2 \to 8=3+9\cos\theta-\cos\theta-3\cos^2\theta \to$

$5=8\cos\theta-3\cos^2\theta \to -3\cos^2\theta+8\cos\theta-5=0 \to \cos\theta=\frac{5}{3}$ غ ق ق

$\boxed{\cos\theta=1} \to$



$\to a^2=25-24\times\frac{1}{2}=\boxed{13}$