شماره ۴

رادمهر معتمدی

Date ______ Subject ______

۱)

$OA = OT = 1$

$\angle AOT = \frac{\pi}{2} - \alpha \longrightarrow TB = \tan(\frac{\pi}{2} - \alpha) = \cot(\alpha)$

$OB = \sqrt{OT^2 + TB^2} = \sqrt{1 + \cot^2\alpha} = \frac{1}{\sin\alpha} \xrightarrow[OA=1]{} AB = \frac{1 - \sin\alpha}{\sin\alpha}$

۲)

$\hat{ADB} \longrightarrow \frac{\sin 45^\circ}{6} = \frac{\sin(\pi - \alpha)}{AB} = \frac{\sin\alpha}{AB}$

$\hat{ADC} \longrightarrow \frac{\sin 30^\circ}{3} = \frac{\sin\alpha}{AC}$

$\sin\alpha = \frac{AB\sqrt{2}}{12} = \frac{AC}{6}$

$\Rightarrow \frac{AB}{AC} = \sqrt{2}$

۳)

$A \left| \begin{matrix} \frac{1}{2} \\ ? \end{matrix} \right. \xrightarrow[\text{ناحیه اول}]{x^2+y^2=1} A \left| \begin{matrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{matrix} \right. : \theta_A = 60^\circ$

$B \left| \begin{matrix} ? \\ \frac{1}{2} \end{matrix} \right. \xrightarrow[\text{ناحیه دوم}]{x^2+y^2=1} B \left| \begin{matrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{matrix} \right. : \theta_B = 150^\circ$

$C \left| \begin{matrix} 0 \\ -1 \end{matrix} \right. : \theta_C = 270^\circ$

$\overset{\frown}{AB} = 90$

$\overset{\frown}{BC} = 120^\circ$

$\overset{\frown}{AC} = 150^\circ$

$BC = 2\sin(\frac{120^\circ}{2}) = \sqrt{3}$

رابطه cos برای در ضلع ۱.۲ : $\sqrt{R^2 + R^2 - 2R^2\cos\theta} = R\sqrt{2 - 2\cos\theta} = R\sqrt{4\sin^2\frac{\theta}{2}} = 2R\sin\frac{\theta}{2}$

$AB = 2\sin(\frac{90}{2}) = \sqrt{2}$, $AC = 2\sin(\frac{150^\circ}{2}) = \frac{\sqrt{6} + \sqrt{2}}{2}$

محیط: $2P = \frac{\sqrt{6}+\sqrt{2}}{2} + \sqrt{2} + \sqrt{3} \longrightarrow$ مساحت =

$S = \frac{1}{2} ab \sin\frac{\overset{\frown}{AC}}{2} = \frac{1}{2}(\sqrt{3})(\sqrt{2})(\frac{\sqrt{6}+\sqrt{2}}{4}) = \frac{6+2\sqrt{3}}{8}$

$S = \frac{3+\sqrt{3}}{4}$

۴)

$AC = \log_3 16 = 4\log_3 2$, $AB = \log_2 27 = 3\log_2 3$

$S = \frac{1}{2} AC \cdot AB \sin\hat{A} = \frac{1}{2}(4\log_3 2)(3\log_2 3)(\frac{1}{2}) = 3$

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۵)

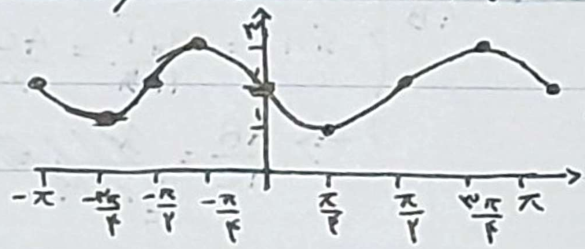
$$f(x) = 2x^4 - x^2 - \sin^2(x)(2\sin^2(x)-1)$$

$$\longrightarrow f(\cos\alpha) = \cancel{2}\cos^2\alpha(\underbrace{2\cos^2\alpha-1}_{\cos 2\alpha}) + \cancel{2}\sin^2(\alpha)(\overbrace{\underbrace{\cancel{\quad}}_{\cos 2\alpha}}^{2\sin^2(\alpha)-1}) = \cos 2\alpha$$

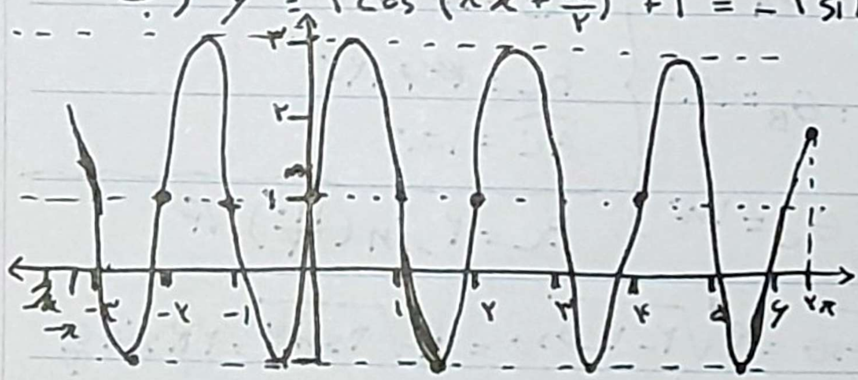
$$f(\cos\alpha) = \cos 2\alpha = 1 \longrightarrow \sin(2\alpha) = 0$$

۶)

الف) $\Rightarrow y = -2\sin x\cos x + 2 = -\sin(2x) + 2 \longrightarrow T = \left|\frac{2\pi}{b}\right| = \pi$

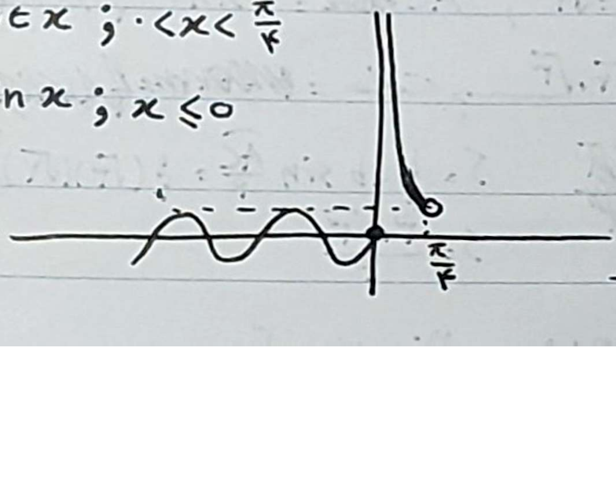


ب) $y = 2\cos(\pi x + \frac{\pi}{2}) + 1 = -2\sin(\pi x) + 1 \longrightarrow T = \left|\frac{2\pi}{b}\right| = 2$



۷)

$$f(x) = \begin{cases} \cot x \ ; \ 0 < x < \frac{\pi}{4} \\ \sin x \ ; \ x \leq 0 \end{cases}$$



$\Longrightarrow R_f = [-1, +\infty)$

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$$f(x) = a + b\,\underbrace{\underbrace{\sin(cx)\cos(cx)}_{\frac{۱}{۲}\sin(۲cx)}\cos(۲cx)}_{\frac{۱}{۴}\sin(۴cx)} = a + \frac{b}{۴}\sin(۴cx)$$

شکل $\Rightarrow T = \frac{\pi}{۲} - \frac{\pi}{۱۰} = \frac{۲\pi}{۵} = \left|\frac{۲\pi}{۴c}\right| \xrightarrow[c>۰]{} c = \frac{۵}{۴}$

شکل $\Rightarrow$ $\left.\begin{array}{l} \max: a + \left|\frac{b}{۴}\right| = ۴ \\ a - \left|\frac{b}{۴}\right| = ۰ \end{array}\right\} \begin{array}{l} |b| = ۸ \\ a = ۲ \end{array}$

$$f(x) = ۲ + \left(\frac{b}{۴}\right)\sin(۵x) \longrightarrow f\left(\frac{\pi}{۱۰}\right) = ۲ + \frac{b}{۴} = ۴ \rightarrow b = +۸$$

$$\rightarrow \frac{bc}{a} = \frac{۸ \times \frac{۵}{۴}}{۲} = ۵$$

(۹

$$f(x) = a\cos(bx) + c \longrightarrow T = \left|\frac{۲\pi}{b}\right| = \pi \rightarrow |b| = ۲$$

شکل : $\left.\begin{array}{l} |a| + c = ۲ \\ -|a| + c = -۴ \end{array}\right\} \begin{array}{l} c = -۱ \\ |a| = ۳ \xrightarrow[\text{شکل}]{a<۰} a = -۳ \end{array}$

$$\rightarrow f(x) = -۳\cos(۲x) - ۱ \rightarrow \cos(۲\alpha) = -\frac{۱}{۳}$$

$$\tan^۲(۲\alpha) = \frac{۱}{\cos^۲(۲\alpha)} - ۱ = ۸ \rightarrow |\tan(۲\alpha)| = ۲\sqrt{۲}$$

(۱۰

$$f(-۳۴/۵) \underset{T=۳}{=} f(۳۶ - ۳۴/۵) = f(۱/۵)$$

شکل $\Rightarrow f(x) = \begin{cases} ۱ - \frac{x}{۲} & ؛ ۰ \le x < ۲ \\ ۲x + ۲ & ؛ -۱ \le x < ۰ \end{cases} \rightarrow f(۱/۵) = ۱ - \frac{۳}{۴} = \frac{۱}{۴}$