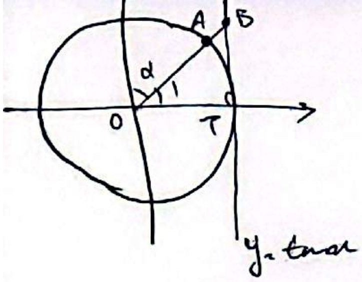
۱-



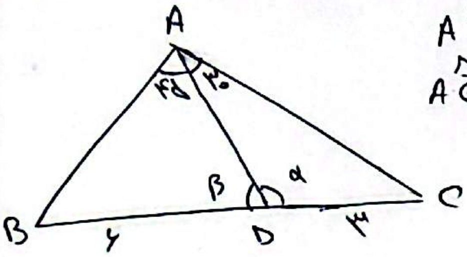
زاویه ۱

$$\text{مثلث}:\ \frac{\sin\hat{1}}{\cancel{BT}} = \frac{\overset{1}{\cancel{OA}}}{OB},\quad BT = \tan\hat{1} \Rightarrow OB = \frac{\tan 1}{\sin 1} = \frac{\frac{\sin 1}{\cos 1}}{\sin 1} \boxed{= \frac{1}{\cos 1} = OB}$$

$$\alpha + \hat{1} = ۹۰^\circ \Rightarrow \boxed{\cos 1 = \sin\alpha} \Rightarrow OB = \frac{1}{\sin\alpha}$$

$$\to AB = OB - OA \xrightarrow{OA = r = 1} AB = \frac{1}{\sin\alpha} - 1 \boxed{= \frac{1-\sin\alpha}{\sin\alpha}}$$

۲-



$$A\overset{\Delta}{B}D:\ \frac{۶}{\sin ۴۵} = \frac{AB}{\sin\beta} = \frac{AD}{\sin B}$$

$$A\overset{\Delta}{C}D:\ \frac{۳}{\sin ۳۰} = \frac{AD}{\sin C} = \frac{AC}{\sin\alpha}$$

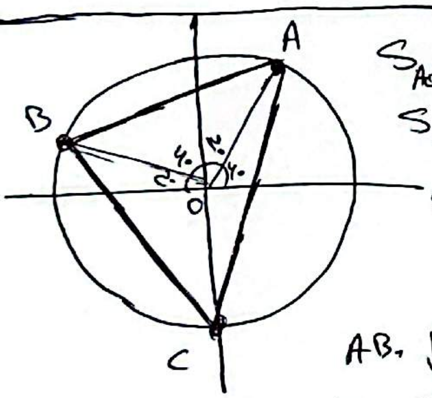
α, β مکمل هستند
پس $\sin\beta = \sin\alpha$

$$AB = \frac{۶\sin\beta}{\cancel{\sin ۴۵}_{\frac{\sqrt{۲}}{۲}}} = \frac{۱۲\sin\beta}{\sqrt{۲}}$$

$$AC = \frac{۳\sin\alpha}{\cancel{\sin ۳۰}_{\frac{1}{۲}}} = ۶\sin\alpha$$

$$\frac{AB}{AC} = \frac{\frac{۱۲\sin\beta}{\sqrt{۲}}}{۶\sin\alpha} = \frac{\overset{۲}{\cancel{۱۲}}\cancel{\sin\alpha}}{\cancel{۶}\sqrt{۲}\cancel{\sin\alpha}} = \frac{۲}{\sqrt{۲}} \boxed{= \sqrt{۲}}$$

۳- الف)



$$S_{AOB} = 1\times1\times\frac{1}{۲}\times\sin ۹۰ = \boxed{\frac{1}{۲}}$$

$$S_{AOC} = 1\times1\times\frac{1}{۲}\times\cancel{\sin ۱۵۰}_{\frac{1}{۲}} = \boxed{\frac{1}{۴}}$$

$$S_{BOC} = 1\times1\times\frac{1}{۲}\times\cancel{\sin ۱۲۰}_{\frac{\sqrt{۳}}{۲}} = \boxed{\frac{\sqrt{۳}}{۴}}$$

$$S_T = S_{ABC} \boxed{= \frac{۳+\sqrt{۳}}{۴}}$$

ب)

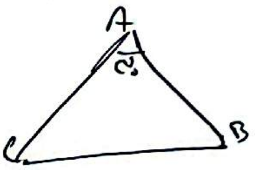
$$AB = \sqrt{1+1-۲\cos ۹۰} = \sqrt{۲}$$

$$AC = \sqrt{1+1-۲\cos ۱۵۰} = \sqrt{۲+\sqrt{۳}}$$

$$BC = \sqrt{1+1-۲\cos ۱۲۰} = \sqrt{۳}$$

محیط $= \sqrt{۲} + \sqrt{۲+\sqrt{۳}} + \sqrt{۳}$

۴-



$$AB = \log_{۲}^{۲۷} = ۳\log_{۲}^{۳},\quad AC = \log_{۳}^{۱۶} = ۴\log_{۳}^{۲}$$

$$S = \frac{1}{۲}\times AC\times AB\times\cancel{\sin ۳۰}_{\frac{1}{۲}} = \frac{1}{۴}\times ۳\log_{۲}^{۳}\times ۴\log_{۳}^{۲} = \frac{1}{۴}\times\cancel{۳\log_{۲}^{۳}}\times\frac{۴}{\cancel{\log_{۲}^{۳}}}$$

$$= \frac{1}{۴}\times ۱۲ = \boxed{۳}$$

۵-

$$۲\cos^۴\alpha - \cos\alpha^۲ - \sin^۲\alpha\times\underbrace{(۲\sin^۲\alpha - 1)}_{\sin^۲ - \cos^۲} = 1 \Rightarrow ۲\cos^۴ - \cos^۲ - \sin^۴ + \sin^۲\cos^۲ = 1$$

$$\Rightarrow \underbrace{\cos^۲(\underbrace{\sin^۲ - 1}_{-\cos^۲})}_{-\cos^۴} + ۲\cos^۴ - \sin^۴ = 1 \Rightarrow -\cos^۴ + ۲\cos^۴ - \sin^۴ = 1 \Rightarrow \cos^۴ - \sin^۴ = 1$$

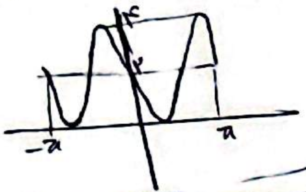
$$\Rightarrow (\cos^۲ - \sin^۲)\underbrace{\cancel{(\cos^۲ + \sin^۲)}}_{1} = 1 \Rightarrow \cos^۲ - \sin^۲ = 1 \Rightarrow \boxed{\cos ۲\alpha = 1}$$

$$\cos ۲\alpha = 1 \Rightarrow ۲\alpha = ۲k\pi \Rightarrow \boxed{\sin ۲\alpha = 0}$$

کاظم الهی

۶-

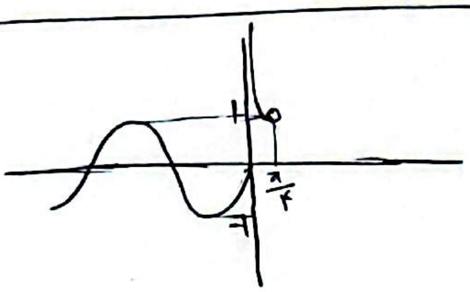
الف) $y = -2\sin x\cos x + 2 = -\sin 2x + 2 \quad , \quad T = \frac{2\pi}{2} = \pi$



ب) $y = 2\cos(x + \frac{1}{2})\pi + 1$

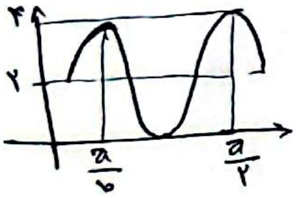
۷-

$f(x)\begin{cases} \cot x & , 0 < x < \frac{\pi}{4} \\ \sin x & , x \leq 0 \end{cases}$



$R_f = (-1, +\infty)$

۸-



$f(x) = \underset{\downarrow}{a} + \underbrace{b\sin(cx) \times \cos(cx)} \times \underset{\downarrow}{\cos(2xc)}$

$a + \frac{b}{2}\underbrace{\sin(2cx) \times \cos(2cx)}$

$a + \frac{b}{2} \times \frac{1}{2} \times \sin(4cx) \Rightarrow f(x) = a + \frac{b}{4}\sin(4cx)$

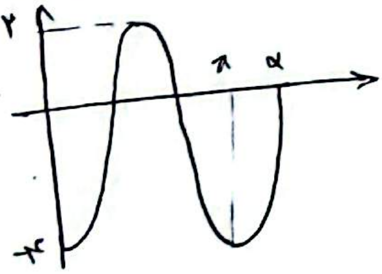
$T = \frac{\pi}{2} - \frac{\pi}{10} = \frac{2\pi}{5} = \frac{2\pi}{4c} \rightarrow 4c = 5 \rightarrow \boxed{c = \frac{5}{4}}$

انبساط طولی $\times 2 \rightarrow \frac{b}{4} = 2 \rightarrow \boxed{b = 8}$

دو واحد بالا $\rightarrow \boxed{a = 2}$

$\frac{bc}{a} = \frac{8 \times 5}{4 \times 2} = \boxed{5}$

۹-



$f(x) = a\cos bx + c$

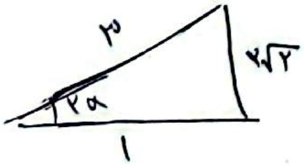
$T = \pi = \frac{2\pi}{b} \rightarrow \boxed{b = 2}$

$\boxed{a = -3}$

$\boxed{c = -1}$

$\begin{cases} f(x) = -3\cos 2x - 1 \\ f(\alpha) = 0 \rightarrow -3\cos 2\alpha - 1 = 0 \end{cases}$

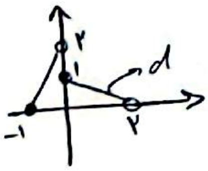
$\rightarrow -3\cos 2\alpha = 1 \rightarrow \boxed{\cos 2\alpha = -\frac{1}{3}}$



$\Rightarrow |\tan 2\alpha| = \frac{2\sqrt{2}}{1} = \boxed{2\sqrt{2}}$

۱۰-

$T = 3 \Rightarrow f(-34{,}5) = f(-37 + 2{,}5) = f(-1 + 2{,}5) = \boxed{f(1{,}5)}$



$d: -\frac{x}{2} + 1 \Rightarrow f(1{,}5) = -\frac{3}{4} + 1 = \boxed{\frac{1}{4}}$

طاهر ایجی