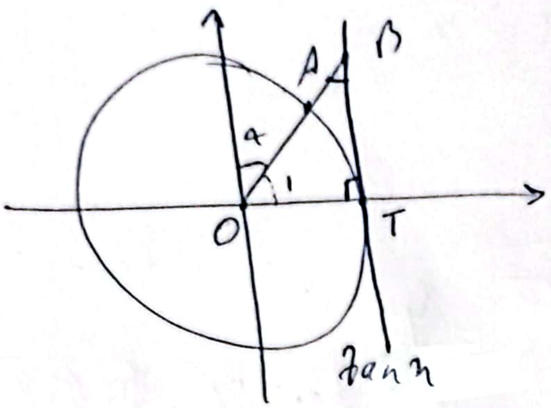
۱۹,۵ آفرین

بنام خدا

ماهان نژادکرد — دوازدهم بسر B — تکلیف ۱٤

۱- ۲



$\cos\hat{O}_1 = \frac{OT}{OB} = \frac{1}{OB}$

$\Rightarrow OB = \frac{1}{\cos\hat{O}_1}$

$\hat{O}_1 + \alpha = 90 \Rightarrow \cos\hat{O}_1 = \sin\alpha$ $\Big\} \; OB = \frac{1}{\sin\alpha}$

$AB = OB - OA = \frac{1}{\sin\alpha} - 1 = \boxed{\frac{1-\sin\alpha}{\sin\alpha}}$

۲- ۲

$S_{A\hat{D}C} = \frac{1}{2} \times AD \times DC \times \sin\alpha$

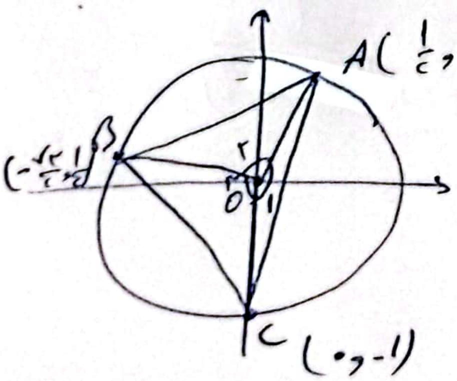
$S_{B\hat{D}A} = \frac{1}{2} \times AD \times BD \times \sin(\pi - \alpha) = \frac{1}{2} AD \cdot BD \sin\alpha$

$\Rightarrow \frac{S_{A\hat{D}C}}{S_{B\hat{D}A}} = \frac{CD}{BD} = \frac{3}{6} = \frac{1}{2}$

$\frac{S_{A\hat{D}C}}{S_{A\hat{B}D}} = \frac{\frac{1}{2} \times AD \times AC \times \sin 30}{\frac{1}{2} \times AD \times AB \times \sin 45} = \frac{\frac{1}{2} AC}{\frac{\sqrt{2}}{2} AB} = \frac{1}{2}$

$\Rightarrow \frac{AB}{AC} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$

۳- الف) با توجه به شکل داریم: ۲



A	$\frac{1}{2}$, $\frac{\sqrt{3}}{2}$
B	$-\frac{\sqrt{3}}{2}$, $\frac{1}{2}$
C	0, -1

۳:

$C\hat{O}A = 90 + 60 = 150 \Rightarrow S_{C\hat{O}A} = 1 \times 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$A\hat{O}B = 90^\circ \Rightarrow S_{A\hat{O}B} = 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$

$B\hat{O}C = 120^\circ \Rightarrow S_{B\hat{O}C} = 1 \times 1 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$\Rightarrow S_T = S_{ABC} = \frac{3 + \sqrt{3}}{4}$

ب)

$AC = \sqrt{OA^2 + OC^2 - 2\,OA\,OC\cos 150^\circ} = \sqrt{2 + \sqrt{3}}$

$AB = \sqrt{OA^2 + OB^2 - 2\,OA\,OB\cos 90} = \sqrt{2}$

$BC = \sqrt{OB^2 + OC^2 - 2\,OB\,OC\cos 120} = \sqrt{3}$

محیط: $2P = \sqrt{2} + \sqrt{3} + \sqrt{2 + \sqrt{3}}$

۴-

$S = \frac{1}{2} \times AB \times AC \sin\alpha = \frac{1}{4} \times \log_2 27 \times \log_3 16$

$= \frac{1}{4} \times 3\log_2 3 \times 4\log_3 2 = 3\log_2 3 \times \frac{1}{\log_2 3} = 3$

$* \log_b a^n = n\log_b a \qquad * \log_b a = \frac{1}{\log_a b}$

۵-

$f(x) = 2x^4 - x^2 - (\sin^2\alpha)(2\sin^2\alpha - 1)$

$\Rightarrow f(\cos\alpha) = 2\cos^4\alpha - \cos^2\alpha - (\sin^2\alpha)(2\sin^2\alpha - 1) = 1$

$= \cos^2\alpha(2\cos^2\alpha - 1) - 2\sin^4\alpha + \sin^2\alpha = 1$

$= \cos^2\alpha(\cos 2\alpha) - \sin^2\alpha(\underbrace{2\sin^2\alpha - 1}_{-\cos 2\alpha}) = 1$

$= \cos^2\alpha(\cos 2\alpha) + \sin^2\alpha(\cos 2\alpha) = \cos 2\alpha(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}) = 1$

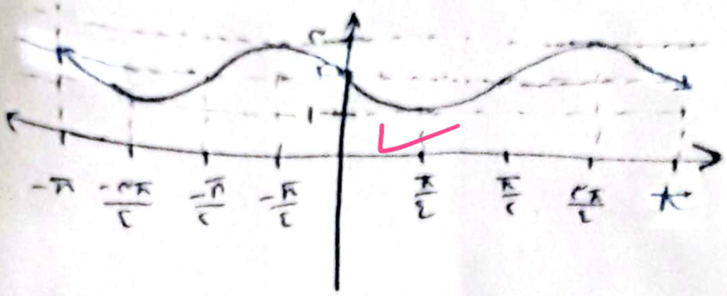
$\Rightarrow \cos 2\alpha = 1 \Rightarrow \sin^2 2\alpha = 1 - \cos^2 2\alpha = 0$

$\Rightarrow \boxed{\sin 2\alpha = 0}$

$y = -2\sin x \cos x + 2 = -\sin 2x + 2$

(۱,۵)

$T = \frac{2\pi}{|a|} = \frac{2\pi}{2} = \pi$



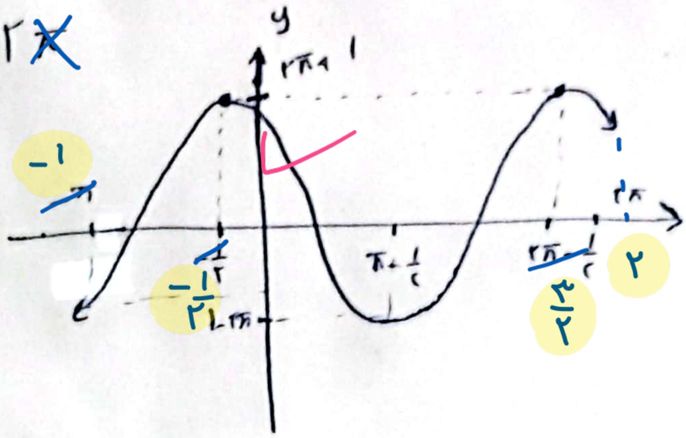
ب) $y = 2\cos(x + \frac{1}{2})\pi + 1 = 2\pi\cos(x + \frac{1}{2}) + 1$ دقت کن !!

$T = \frac{2\pi}{|a|} = \frac{2\pi}{1} = 2\pi$

$2\cos(\pi x + \frac{\pi}{2}) + 1$

$-2\sin(\pi x) + 1$

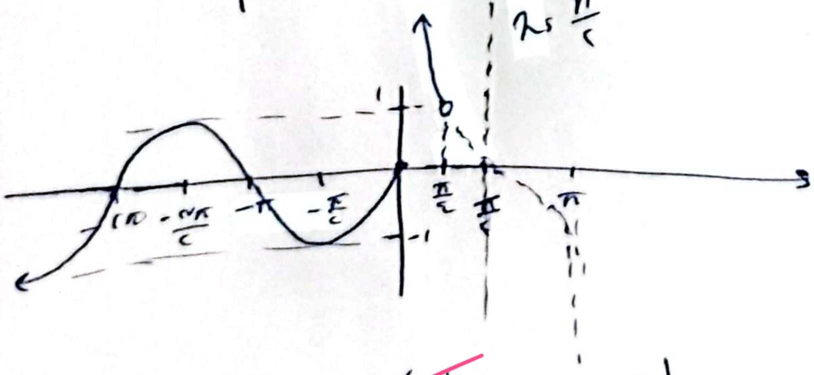
$\frac{2\pi}{|\pi|} = 2$



-۷

$f(x) = \begin{cases} \cot x & ; \; 0 < x < \frac{\pi}{2} \\ \sin x & ; \; x \le 0 \end{cases}$

(۲)



$R_f = [-1, +\infty)$

۸- (۳)

$f(x) = a + b\sin(cx)\cos(cx)\cos(2cx)$

$= a + b \times \frac{1}{2}\sin(2cx)\cos(2cx) = a + b \times \frac{1}{4}\sin(4cx) = a + \frac{b}{4}\sin(4cx)$

$T = \frac{\pi}{2} - \frac{\pi}{10} = \frac{4\pi}{10} = \frac{2\pi}{5} = \frac{2\pi}{|4c|} \Rightarrow 4c = 5 \Rightarrow c = \frac{5}{4}$, $c > 0$

[illegible] به شکل !

$\rightarrow f(x) = a + \frac{b}{2}\sin 2x$

$\max f(x) = a + \frac{|b|}{2} = 4 \quad \Rightarrow a = 2$

$\min f(x) = a - \frac{|b|}{2} = 0 \quad \Rightarrow |b| = 4$

چون نمودار در بازه $[0, \frac{\pi}{4}]$ صعودی است پس $b > 0$ پس $b = 4$

$\rightarrow \frac{bc}{a} = \frac{4\times\frac{\pi}{2}}{2} = \pi$

۹- $y = a\cos(bx) + c$

$T = \pi = \frac{2\pi}{|b|} \rightarrow |b| = 2 \rightarrow b = \pm 2$

چون نمودار در بازه $[0, \frac{\pi}{2}]$ صعودی است پس $a < 0$ و داریم:

$\begin{cases} |a| + c = 2 \\ -|a| + c = -4 \end{cases} \Rightarrow c = -1$

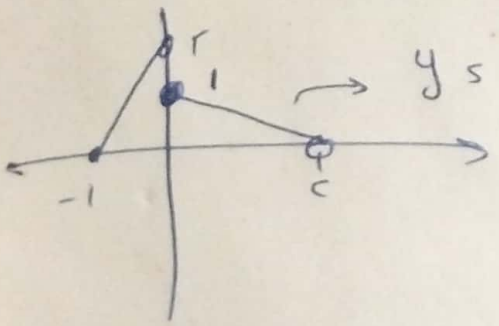
$|a| = 3 \rightarrow a = -3$

$\rightarrow f(x) = -3\cos(2x) - 1$

$\Rightarrow -3\cos(2\alpha) - 1 = 0 \rightarrow \cos(2\alpha) = \frac{1}{3} \rightarrow \tan^2 2\alpha = \frac{1}{\cos^2 2\alpha} - 1$

$\rightarrow \tan^2 2\alpha = \frac{1}{\frac{1}{9}} - 1 = 8 \rightarrow |\tan 2\alpha| = 2\sqrt{2}$

۱۰- $T = 2 \rightarrow f(-3\frac{1}{2}) = f(-3\frac{1}{2} + 2) = f(\frac{1}{2})$



$\rightarrow y = -\frac{1}{2}x + 1 \rightarrow f(\frac{1}{2}) = -\frac{1}{2}\times\frac{1}{2} + 1 = \frac{-1}{4} + 1 = \frac{3}{4}$