به نام خدا

ماهان نژادکرد — دوازدهم بیسر B — تکلیف ۱۴

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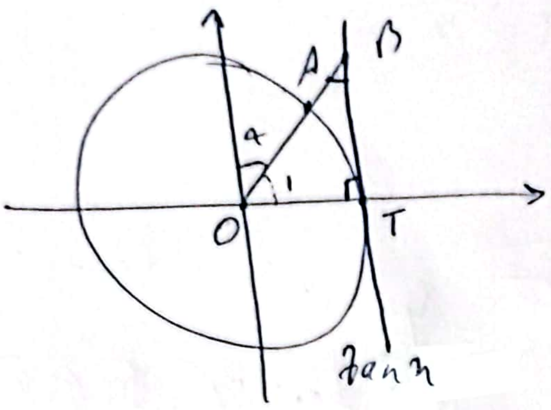


$$\cos\hat{O}_1 = \frac{OT}{OB} = \frac{1}{OB}$$

$$\Rightarrow OB = \frac{1}{\cos\hat{O}_1}$$

$$\hat{O}_1 + \alpha = 90 \Rightarrow \cos\hat{O}_1 = \sin\alpha \quad \Big\} \quad OB = \frac{1}{\sin\alpha}$$

$$AB = OB - OA = \frac{1}{\sin\alpha} - 1 = \boxed{\frac{1-\sin\alpha}{\sin\alpha}}$$

۲-

$$S_{A\hat{D}C} = \frac{1}{2} \times AD \times DC \times \sin\alpha$$

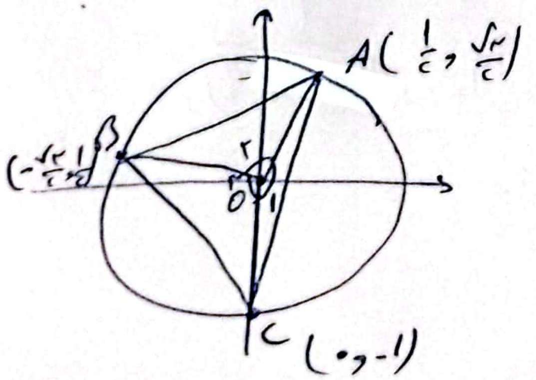
$$S_{B\hat{D}A} = \frac{1}{2} \times AD \times BD \times \sin(\pi - \alpha) = \frac{1}{2} AD . BD \sin\alpha$$

$$\Rightarrow \frac{S_{A\hat{D}C}}{S_{B\hat{D}A}} = \frac{CD}{BD} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{S_{A\hat{D}C}}{S_{A\hat{B}D}} = \frac{\frac{1}{2} \times AD \times AC \cdot \sin 30}{\frac{1}{2} \times AD \times AB \cdot \sin 45} = \frac{\frac{1}{2} AC}{\frac{\sqrt{2}}{2} AB} = \frac{1}{2}$$

$$\Rightarrow \frac{AB}{AC} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$$

۳- الف) با کمک شکل داریم:



$$A \left| \begin{array}{l} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{array}\right. \qquad B \left| \begin{array}{l} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{array}\right. \qquad C \left| \begin{array}{l} 0 \\ -1 \end{array}\right.$$

ادامه:

$$C\hat{O}A = 90 + 60 = 150 \Rightarrow S_{C\hat{O}A} = 1 \times 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$A\hat{O}B = 90^\circ \Rightarrow S_{A\hat{O}B} = 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$$

$$B\hat{O}C = 120^\circ \Rightarrow S_{B\hat{O}C} = 1 \times 1 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$\Rightarrow S_T = S_{ABC} = \frac{3+\sqrt{3}}{4}$$

ب)

$$AC = \sqrt{OA^2 + OC^2 - 2OA\,OC\cos 150^\circ} = \sqrt{2+\sqrt{3}}$$

$$AB = \sqrt{OA^2 + OB^2 - 2OA\,OB\cos 90} = \sqrt{2}$$

$$BC = \sqrt{OB^2 + OC^2 - 2OB\,OC\cos 120} = \sqrt{3}$$

محیط $= 2P = \sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}$

۴ـ

$$S = \frac{1}{2} \times AB \times AC \sin\alpha = \frac{1}{4} \times \log_2 27 \times \log_3 16$$

$$\overset{*}{=} \frac{1}{4} \times 3\log_2 3 \times 4\log_3 2 \overset{*}{=} 3\log_2 3 \times \frac{1}{\log_2 3} = 3$$

$$* \log_b a^n = n\log_b a \qquad * \log_b a = \frac{1}{\log_a b}$$

۵ـ

$$f(x) = 2x^4 - x^2 - (\sin^2\alpha)(2\sin^2\alpha - 1)$$

$$\Rightarrow f(\cos\alpha) = 2\cos^4\alpha - \cos^2\alpha - (\sin^2\alpha)(2\sin^2\alpha - 1) = 1$$

$$= \cos^2\alpha(2\cos^2\alpha - 1) - 2\sin^4\alpha + \sin^2\alpha = 1$$

$$= \cos^2\alpha(\cos 2\alpha) - \sin^2\alpha\underbrace{(2\sin^2\alpha - 1)}_{-\cos 2\alpha} = 1$$

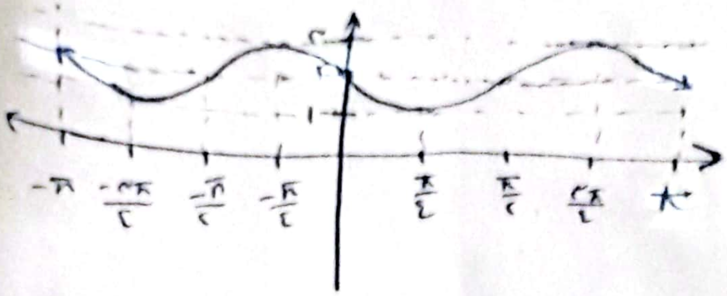
$$= \cos^2\alpha(\cos 2\alpha) + \sin^2\alpha(\cos 2\alpha) = \cos 2\alpha\underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1} = 1$$

$$\Rightarrow \cos 2\alpha = 1 \Rightarrow \sin^2 2\alpha = 1 - \cos^2 2\alpha = 0$$

$$\Rightarrow \boxed{\sin 2\alpha = 0}$$

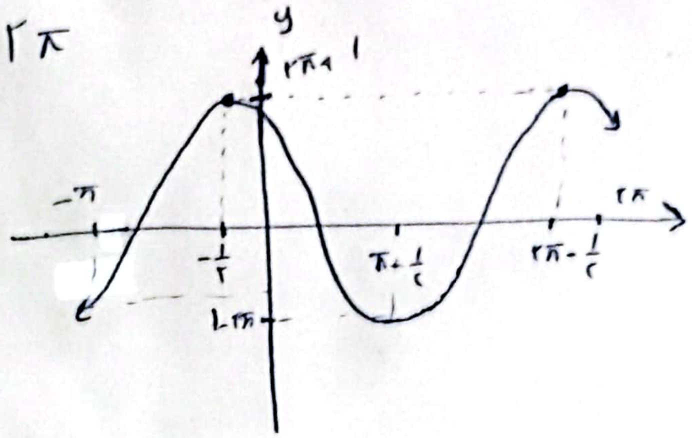
$$y = -2\sin x\cos x + 2 = -\sin 2x + 2$$ ۶ -

$$T = \frac{2\pi}{|a|} = \frac{2\pi}{2} = \pi$$



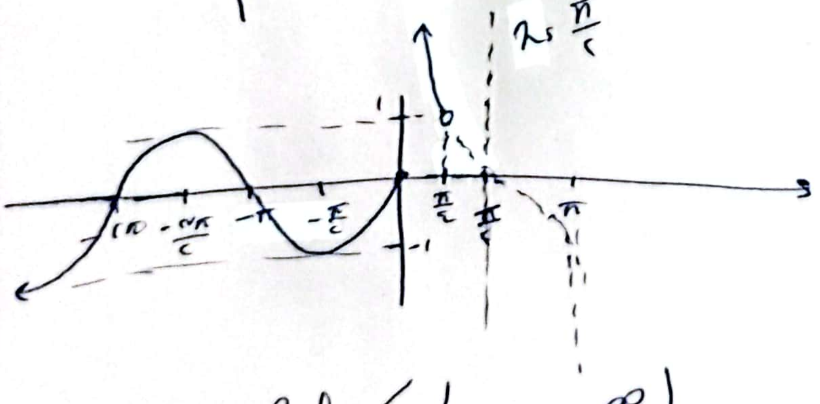
ب) $y = 2\cos(x - \frac{1}{2})\pi + 1 = 2\pi\cos(x - \frac{1}{2}) + 1$

$$T = \frac{2\pi}{|a|} = \frac{2\pi}{1} = 2\pi$$



۷ -

$$f(x) = \begin{cases} \cot x & ; \; 0 < x < \frac{\pi}{2} \\ \sin x & ; \; x \le 0 \end{cases}$$



$$R_f = [-1, +\infty)$$

۸ -

$$f(x) = a + b\sin(cx)\cos(cx)\cos(2cx)$$

$$= a + b \times \frac{1}{2}\sin(2cx)\cos(2cx) = a + b \times \frac{1}{4}\sin(4cx) = a + \frac{b}{4}\sin(4cx)$$

$$T = \frac{\pi}{2} - \frac{\pi}{10} = \frac{4\pi}{10} = \frac{2\pi}{5} = \frac{2\pi}{|4c|} \Rightarrow 4c = 5 \Rightarrow c = \frac{5}{4}, \; c > 0$$

آلو چه مشکل !

$\to f(x) = a + \frac{b}{4} \sin 5x$

$$\max f(x) = a + \frac{|b|}{4} = 4 \quad ; \quad a = 2$$
$$\min f(x) = a - \frac{|b|}{4} = 0 \quad ; \quad |b| = 8$$

چون نمودار در بازه $(0, \frac{\pi}{5}]$ صعودی است پس b مثبت است پس $|b| = b = 8$

$$\to \frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = 5$$

۹ـ $y = a\cos(bx) + c$

$$T = \pi = \frac{2\pi}{|b|} \to |b| = 2 \to b = \pm 2$$

با توجه به شکل داریم:

چون نمودار در بازه $(0, \frac{\pi}{4}]$ صعودی است پس $a<0$ داریم:

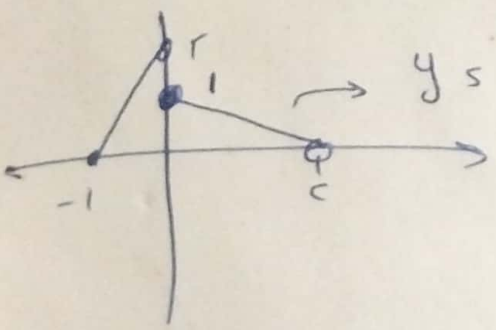
$$\begin{cases} |a| + c = 2 \\ -|a| + c = -4 \end{cases} \Rightarrow c = -1$$
$$|a| = 3 \to a = -3$$

$$\to f(x) = -3\cos(2x) - 1$$

$$\Rightarrow -3\cos(2\alpha) - 1 = 0 \to \cos(2\alpha) = \frac{1}{3} \to \tan^2 2\alpha = \frac{1}{\cos^2 2\alpha} - 1$$

$$\to \tan^2 2\alpha = \frac{1}{\frac{1}{9}} - 1 = 8 \to |\tan 2\alpha| = 2\sqrt{2}$$

۱۰ـ $T = 3 \Rightarrow f(-345) = f(-345 + 112(3)) = f(15)$



$$\to y = -\frac{1}{2}x + 1 \to f(\frac{3}{2}) = -\frac{1}{2} \times \frac{3}{2} + 1 = \frac{-3}{4} + 1 = \frac{1}{4}$$