$BT = \tan(\frac{\pi}{4} - \alpha) \cdot \cot\alpha \to OB = \sqrt{1 + \cot^2\alpha} = \frac{1}{\sin\alpha} \to AB = OB - OA$

$\frac{1}{\sin\alpha} - 1 = \frac{1 - \sin\alpha}{\sin\alpha}$ ①

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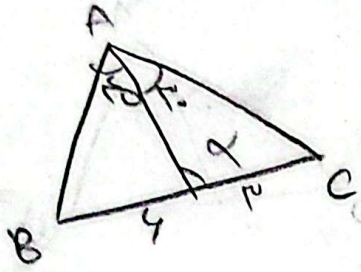
④



$\frac{۳}{\sin۴۵} = \frac{AC}{\sin\alpha} \to AC = ۴\sin\alpha$

$\frac{۴}{\sin۶۰} = \frac{AB}{\sin(\pi - \alpha)} \to AB = ۴\sqrt{۲}\sin\alpha$

$\to \frac{AB}{AC} = \frac{۴\sqrt{۲}\sin\alpha}{۴\sin\alpha} = \sqrt{۲}$

---

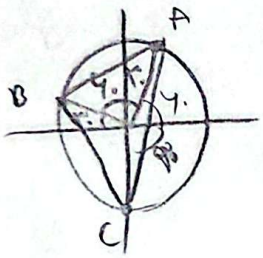
③

$A: (\frac{۱}{۲}, \frac{\sqrt{۳}}{۲})$

$B: (-\frac{\sqrt{۳}}{۲}, \frac{۱}{۲})$

$C: (۰, -۱)$

$S = \frac{۱}{۲}\left|\begin{matrix} \frac{۱}{۲} & \frac{\sqrt{۳}}{۲} & ۱ \\ -\frac{\sqrt{۳}}{۲} & \frac{۱}{۲} & ۱ \\ ۰ & -۱ & ۱ \end{matrix}\right| = \left(\frac{۱}{۴} + \frac{\sqrt{۳}}{۴} - \left(-\frac{۳}{۴} - \frac{۱}{۲}\right)\right) \times \frac{۱}{۲} = \frac{۳ + \sqrt{۳}}{۴}$ الف



$AB = ۲R\sin۶۰ = ۴ \times ۱ \times \frac{\sqrt{۳}}{۲} = ۲\sqrt{۳}$

$AC =$

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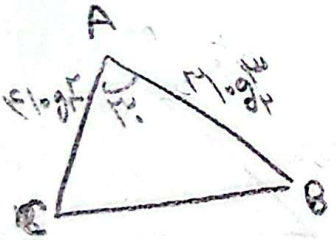
⑤



$S = \frac{۱}{۲} \times \log_۲ ۳ \times \log_۲ ۴ \times \sin۷۵ = \frac{۲\log_۲ ۳}{\log_۲ ۲} = ۲\log_۲ ۳$

---

⑥

$۲\sin^۲x(۲\cos^۲x - ۱) - \sin^۲x(۲\sin^۲x - ۱) = f(x)$

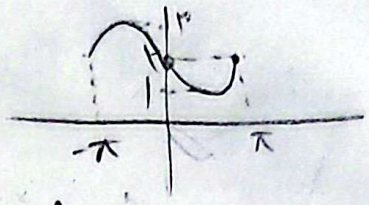
$۲\sin^۲\alpha(۲\cos^۲\alpha - ۱) - \sin^۲\alpha(۲\sin^۲\alpha - ۱) = ۲ - \cos۲\alpha(۹(x)) + ۱$

$f(\cos\alpha) = \underbrace{\cos^۲\alpha(۲\cos^۲\alpha - ۱)}_{\cos۲\alpha} - \sin^۲\alpha\underbrace{(۲\sin^۲\alpha - ۱)}_{-\cos۲\alpha} = \cos۲\alpha\underbrace{(\cos^۲\alpha + \sin^۲\alpha)}_{۱} = ۱ \to \cos۲\alpha = ۱$

$\to \sin۲\alpha = ۰$

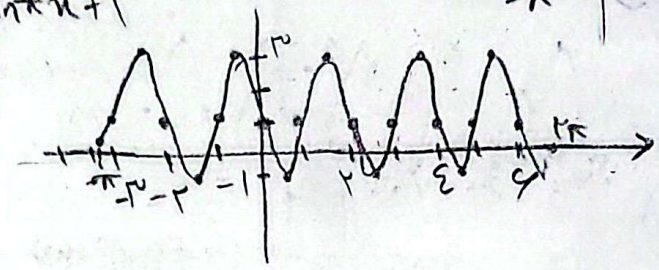
(۴)

ب) $-۲\sin x\cos x+۲=-\sin ۲x+۲$ $\quad T=\frac{۲\pi}{|۲|}=\pi$



ب) $۲\cos(۴\pi x+\frac{\pi}{۲})+۱=-۲\sin ۴\pi x+۱$

$T=\frac{۲\pi}{۸}=\frac{۱}{۴}$



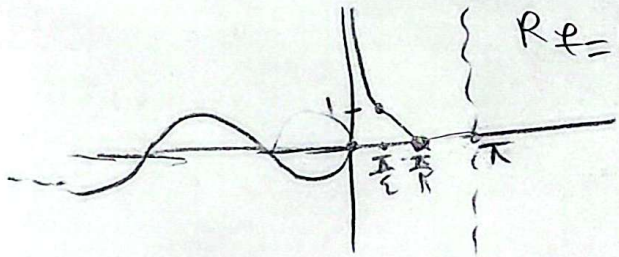
$[-\pi,\pi]\cong[-۲\pi,۶\pi]$

---

(۷)

$$f(x)=\begin{cases}\cot x & ;\ ۰<x<\frac{\pi}{۲}\\ \sin x & ;\ x\le ۰\end{cases}$$

$\cot x=\tan(\frac{\pi}{۲}-x)$



$R_f=(-۱,+\infty)$

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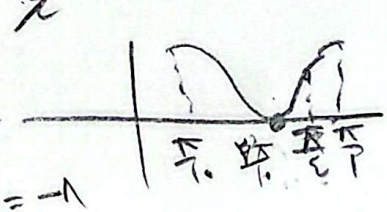
(۸)

$f(x)=a+b\sin(x)\cos(cx)(\cos(۲cx)=a+\frac{b}{۲}\sin(۲cx)\cos(۲cx)=a+\frac{b}{۴}\sin(۴cx)$

$T=\frac{\pi}{۲}-\frac{\pi}{۱۰}=\frac{۵\pi-\pi}{۱۰}=\frac{۴\pi}{۱۰}=\frac{۲\pi}{۵}=\frac{۲\pi}{۴|c|}\to|c|=\frac{۵}{۴}\xrightarrow{c>۰}c=\frac{۵}{۴}$

$a+|\frac{b}{۲}|=۳$
$a-|\frac{b}{۲}|=۰\to a=۲,\ |\frac{b}{۲}|=۲$

$f'(x)=-۴c\frac{b}{۲}\sin ۴cx=bc\sin ۴cx=bc\sin ۵\alpha x$



$f'(\frac{\pi}{۴})>۰\to b\underbrace{\sin\frac{۵\pi}{۴}}_{<۰}>۰\to b<۰\to\frac{b}{۲}=-۲\to b=-۸$

$-\frac{۴\pi}{۲}\times\frac{۵}{۴}=-۵\pi$

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(۹)

$|a|+c=۲$
$-|a|+c=-۲\to c=-۱$
$|a|=۳$
$\frac{۲\pi}{|b|}=\pi\to|b|=۲\to f(x)=-۲\cos ۲x$

$f'(۰)>۰\to a\ne-۳$

$-۱,-۳\cos ۲\alpha-۱=۰$
$\to\cos ۲\alpha=-\frac{۱}{۳}$
$\tan ۲\alpha$
$۱+\tan^۲ ۲\alpha=۹\to\tan ۲\alpha=۲\sqrt{۲}$