۱۷/۷۵

نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۱۵...... کلاس .... B

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$AC = 4$

$S_{ABC} = \frac{1}{2} AC \times OB = \frac{1}{2} \times 4 \times 1 = \frac{\sqrt{3}}{2}$

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$9\sin^2 x - 18\sin x + 7$

$(\sin x - \ldots)(\sin x - 2) \ldots$

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$\log \frac{\sin x}{-\cos x} = \log(-\tan x) = \log 3 \Rightarrow \tan x = -3$

$1 + \tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 1 + 9 = 10 \Rightarrow \cos x = -\frac{\sqrt{10}}{10}$

$\sin x > 0, \quad \cos x < 0$

$\sin x = \frac{3\sqrt{10}}{10}$

$\sin x - \cos x = \frac{3\sqrt{10}}{10} + \frac{\sqrt{10}}{10} = \frac{4\sqrt{10}}{10}$

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$\sin^4\theta = (1 - \cos^2\theta)^2 = 1 + \cos^4\theta - 2\cos^2\theta$

$2\cos^4 x - \cos^2 x + 1 = \frac{4}{5}$

$\cos^4 x + \sin^2 x = \cos^4 x + 1 - \cos^2 x = \frac{4}{5}$

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$\frac{\sin^2 x}{\cos x} - \frac{1}{\cos x} = \frac{\sin^2 x - 1}{\cos x} > 0, \quad \frac{-\cos^2 x}{\cos x} > 0 \Rightarrow \cos x < 0$

$\ldots = 2\sin^2 x > 0$

$\sin x < 0$

در ناحیه سوم

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$\frac{2\cos\alpha + \sin\alpha}{\sin\alpha\cos\alpha} =$ $2\cos\alpha = -\sin\alpha \Rightarrow \tan\alpha = -\frac{1}{2}$

$\cos\alpha = \frac{1}{\sqrt{5}}$

$= \frac{\frac{1}{\sqrt{5}}}{\frac{-4}{5}} = \frac{-5}{4\sqrt{5}} = -\frac{\sqrt{5}}{4}$ (2)

$\sin 2\alpha + \cos 2\alpha = 2\sin\alpha\cos\alpha + \cos^2\alpha - \sin^2\alpha$

$= -\frac{4}{5} + \frac{1}{5} - \frac{4}{5} = -\frac{7}{5}$ (2)

$\frac{\sqrt{1+\sin\omega}}{\sin\frac{\omega}{2} + \sin\frac{\omega}{2}} = \frac{\sqrt{\cos^2\frac{\omega}{2} + \sin^2\frac{\omega}{2} + 2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}}{2\sin\frac{\omega}{2}} = \frac{\cos\frac{\omega}{2} + \sin\frac{\omega}{2}}{2\sin\frac{\omega}{2}} = \frac{1}{2}$ (1)

$\sin\beta + \sin\gamma = 2\sin\frac{\beta+\gamma}{2}\cos\frac{\beta-\gamma}{2}$ (2)

$\frac{\sin 2\alpha}{\sin\alpha} \times \frac{\cos\alpha}{\frac{1}{2}\sin\alpha} \times \frac{\cos 2\alpha}{\frac{1}{4}\sin 2\alpha} \times \cos 4\alpha = \frac{8\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha}{\sin\alpha} = \frac{\sin 8\alpha}{\sin\alpha} = \frac{1}{8}$ (1,5)

$\frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{\frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha}} = \cos 2\alpha \quad \frac{1}{8}\frac{\cos\alpha}{\sin\alpha} = \frac{1}{8}\cot\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \frac{1}{4}\cos^2\alpha = 1$

$\frac{5}{4}\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{2}{\sqrt{5}}$ (2)

$\sin\alpha = \frac{1}{\sqrt{5}}$

$\cos 2\alpha = 2\cos^2\alpha - 1 = \frac{3}{5}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{4}{5}$

$\cos(2\alpha - \beta) = \cos 2\alpha\cos\beta + \sin 2\alpha\sin\beta = \frac{3}{5}\cdot\frac{1}{\sqrt{2}} + \frac{4}{5}\cdot\frac{1}{\sqrt{2}} = \frac{7}{5\sqrt{2}}$ (4)

$\cos^2\left(\frac{B+C}{2}\right) = \sin B \sin A C$

$\frac{1+\cos(B+C)}{2} = \sin B \sin A C$

$1 + \cos B\cos C - \sin B\sin C = 2\sin B\sin C$

$\cos B\cos C + \sin B\sin C = 1$

$\cos(B-C) = 1$

$B = C$ (1)

$x = \frac{2}{3} \rightarrow (\frac{2}{3})^2 - \frac{2}{3}\sin\alpha - \frac{\cos^2\alpha}{4} = 0$ 2

$\frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{(1-\sin^2\alpha)}{4} = 0 \rightarrow \frac{9\sin^2\alpha - 9 - 24\sin\alpha + 16}{36} = 0$

$9\sin^2\alpha - 24\sin\alpha + 7 = 0$ ساده $\rightarrow$ $3\sin^2\alpha - 8\sin\alpha + 7(3) = 0$

$(\sin\alpha - 7)(\sin\alpha - 1) = 0 \rightarrow \sin\alpha = \frac{1}{3}$ ✓ یا ✗ $\frac{7}{3}$

$\sin\alpha = \frac{1}{3} \rightarrow \cos^2\alpha = 1 - \frac{1}{9} = \frac{8}{9}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \boxed{\frac{9}{8}}$

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$\cos^2\frac{A}{2} = \frac{1+\cos A}{2} \rightarrow 2\sin B\sin C = 1 + \cos\hat{A}$ $\xrightarrow{A+B+C=\pi}$ $A = \pi - (\hat{B}+\hat{C})$

$1 + \cos(\pi - (B+C)) = 1 - \cos(B+C) = 2\sin B\sin C$

$1 - \cos B\cos C + \sin B\sin C = 2\sin B\sin C \rightarrow 3\sin B\sin C + \cos B\cos C = 1$

$\cos(B-C) = 1 \rightarrow \begin{cases} B - C = 0 \checkmark \\ B - C = 2\pi \times \text{مثبت} \end{cases} \rightarrow B = C \rightarrow A = 180^\circ - 132^\circ = 48^\circ$

$B + C = 2B$ (2×24)