Date: پانزدهم دی Sub: تکلیف ۱۵ به نام خدا امیرعلی نوروزملکی

۲۰ آفرین

۱)

$y = \tan\left(-2x + \frac{\pi}{4}\right)$ ②

$\Rightarrow x = 0 \Rightarrow y = \tan\frac{\pi}{4} = 1$

$T = \frac{\pi}{|-2|} = \frac{\pi}{2} \Rightarrow AC = \frac{\pi}{2} \Rightarrow S_{ABC} = \frac{\pi}{2} \times 1 \times \frac{1}{2} = \frac{\pi}{4}$ ✓

۲)

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0$

$x = \frac{2}{3} \Rightarrow \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha = 0 \Rightarrow \frac{16 - (24\sin\alpha - 9\cos^2\alpha)}{36} = 0$ ②

$24\sin\alpha + 9(1 - \sin^2\alpha) = 16 \Rightarrow 9\sin^2\alpha - 24\sin\alpha + 7 = 0$

$\Rightarrow (3\sin\alpha - 7)(3\sin\alpha - 1) = 0 \Rightarrow \sin\alpha = \frac{1}{3} \Rightarrow \cos^2\alpha = \frac{8}{9}$

$\Rightarrow 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow \frac{9}{8}$ ✓

۳)

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3$ ②

$\Rightarrow -\frac{\sin\alpha}{\cos\alpha} = 3 \Rightarrow -\tan\alpha = 3 \Rightarrow 1 + \tan^2 = \frac{1}{\cos^2} \Rightarrow \cos^2\alpha = 0.1$

$\sin^2 = \frac{9}{10}$

$\Rightarrow \frac{3\sqrt{10}}{10} + \frac{\sqrt{10}}{10} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$ ✓

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$\sin^4\theta \cos^2\theta = \frac{4}{81} \Rightarrow \sin^4\theta(1-\sin^2\theta) = \frac{4}{81}$ (1)

$\Rightarrow \sin^4\theta - \sin^2\theta + \frac{1}{81} = 0 \Rightarrow 81t^2 - 81t + 1 = 0$ (2)

$t = \frac{81 \pm \sqrt{81}}{162} \Rightarrow \cos^2\theta = 1 - t \Rightarrow \frac{81 - \sqrt{81}}{162}$

$\Rightarrow \cos^4\theta = \frac{2(81) + 81 - 162\sqrt{81}}{162^2} \Rightarrow \frac{324 - 162\sqrt{81}}{162^2} = \frac{2 - \sqrt{81}}{162}$

$\Rightarrow \cos^4\theta + \sin^4\theta \Rightarrow \frac{2 - \sqrt{81}}{162} + \frac{81 + \sqrt{81}}{162} = \frac{\Delta}{162} = 0.1$ (3)

$\frac{2\sin\alpha + \sin\alpha\cos\alpha}{2 - 2\cos^2\alpha} < 0 \Rightarrow \frac{\sin\alpha(2 + \cos\alpha)}{2(1 + \cos^2\alpha)} < 0$ (5)

$\Rightarrow \frac{2 + \cos\alpha}{2\sin\alpha} < 0 \Rightarrow \sin\alpha < 0$ (2)

$\Rightarrow$ (4) Sol

$\frac{\sin^2\alpha}{\cos\alpha} - \frac{1}{\cos\alpha} > 0 \Rightarrow \frac{-\cos^2\alpha}{\cos\alpha} > 0 \Rightarrow \cos\alpha < 0$

$\frac{2}{\sin\alpha} + \frac{2}{\cos\alpha} = 0 \Rightarrow \frac{2\cos\alpha + 2\sin\alpha}{\sin\alpha\cos\alpha} = 0 \Rightarrow 2\sin\alpha = -2\cos\alpha$ (4) (2)

$\Rightarrow 9\sin^2\alpha = 4 - 4\sin^2\alpha \Rightarrow 13\sin^2\alpha = 4 \Rightarrow \sin^2\alpha = \frac{4}{13} \Rightarrow \cos^2\alpha = \frac{9}{13}$

$\Rightarrow \cos\alpha + \cos\alpha = \frac{2}{\sin\alpha\cos\alpha} \Rightarrow \frac{2}{\frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}}} \Rightarrow -\frac{13}{6}$

Elipon

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$$\frac{\sqrt{1+\sin 20^\circ}}{\sin 20^\circ + \sin 10^\circ} \Rightarrow \frac{\sqrt{1+\cos 40^\circ}}{\cos 10^\circ + \cos 40^\circ} \Rightarrow \sqrt{2\cos^2 20^\circ} \Rightarrow \frac{\sqrt{2}\cos 20^\circ}{2\cos^2 40^\circ + \cos 40^\circ - 1}$$

$$\Rightarrow \frac{\sqrt{2}\cos 20^\circ}{(2\cos 40^\circ - 1)(\cos 40^\circ + 1)} \Rightarrow \frac{\sqrt{2}\cos 20^\circ}{(2\cos^2 20^\circ - 2)(2\cos^2 20^\circ)} \Rightarrow \frac{\sqrt{2}}{2} \times \frac{1}{(2\cos^2 - 2\cos 20^\circ)}$$

$$\Rightarrow \frac{\sqrt{2}}{2} \times \frac{1}{\cos 40^\circ} \Rightarrow \frac{\sqrt{2}}{2} \times 2 = \sqrt{2}$$ ✓

(A

$$(1 - 2\sin^2 \alpha)(2\cos^2 10^\circ - 1)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right)$$

$$(\cos 10^\circ)(\cos 20^\circ)(\cos 40^\circ) \times \frac{\sin 10^\circ}{\sin 10^\circ} \Rightarrow \frac{1}{2}\frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ}{\sin 10^\circ}$$

$$\Rightarrow \frac{1}{4}\frac{\sin 40^\circ \cos 40^\circ}{\sin 10^\circ} \Rightarrow \frac{1}{8}\frac{\sin 80^\circ}{\sin 10^\circ} \Rightarrow \frac{1}{8}\frac{\cos 10^\circ}{\sin 10^\circ}$$

$\hookrightarrow \cot 10^\circ$ ✓

(9

$$\sin\alpha + 2\cos\alpha = 2 \Rightarrow \sin\alpha = 2 - 2\cos\alpha$$

$$\Rightarrow \sin^2\alpha = 4 - 8\cos\alpha + 4\cos^2\alpha \Rightarrow 1 = 4 - 8\cos\alpha + 4\cos^2\alpha$$

$$\Rightarrow 4\cos^2\alpha - 8 - 8\cos\alpha = -1 \Rightarrow 4\cos^2\alpha - 8\cos\alpha = -1$$ ✓

Elipon

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$\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{bc}{\sin B \sin C} = \frac{a^2}{\sin^2 A}$

$\frac{\cos^2\frac{A}{2}}{\cos^2\frac{A}{2}} \quad \frac{4\cos^2\frac{A}{2}\sin^2\frac{A}{2}}{\cos^2\frac{A}{2}}$ (2)

$4bc\cdot\sin^2\frac{A}{2} = a^2 \Rightarrow 4bc\left(1-\cos^2\frac{A}{2}\right) = a^2$

$\Rightarrow b^2+c^2-2bc\cos A = 4bc\left(1-\cos^2\frac{A}{2}\right)$ ✓

$b^2+c^2-bc\left(2\cos A+4-4\cos^2\frac{A}{2}\right)$

$\Rightarrow b^2+c^2-bc\left(2(2\cos^2\frac{A}{2}-1)+4-4\cos^2\frac{A}{2}\right)$

$\Rightarrow b^2+c^2-2bc = 0 \Rightarrow (b-c)^2 = 0 \Rightarrow b=c \Rightarrow \hat{B}=\hat{C}$ ✓

$\Rightarrow A = 180 - 144 = 36^\circ$ ✓