

در پاسخنامه ترتیب سوالات کمی به هم ریخته است (لطفاً به شماره سوال نوشته شده توجه فرمایید) با تشکر

نام و نام خانوادگی کلاً ریاضیات پاسخنانه تشریحی تکلیف شماره (ب) کلاس B

۲۰. آفرین ضعیف کمال نوشتی!

$$AC = \frac{1}{\sqrt{2}} \times \pi = \frac{\pi}{\sqrt{2}}$$

$$B = \tan\left(\frac{\pi}{4}\right) = 1$$

$$S_{ABC} = \frac{\pi}{4}$$

۱

$$\textcircled{1} \frac{4}{9} - \frac{2}{3}(\sin \alpha) - \frac{1}{4}(1 - \sin^2 \alpha) = 0 \xrightarrow{\times 12} -9\sin^2 \alpha + 12\sin \alpha - 1 = 0$$

$$\textcircled{2} (3\sin \alpha - 1)(3\sin \alpha - 1) = 0 \Rightarrow \sin \alpha = \frac{1}{3} \Rightarrow \cos \alpha = \frac{2\sqrt{2}}{3}$$

$$\textcircled{3} \frac{1}{\cos^2 \alpha} = 1 + \tan^2 \alpha \Rightarrow \frac{1}{\frac{4}{9}} = 1 + \tan^2 \alpha \Rightarrow \boxed{1 + \tan^2 \alpha = \frac{9}{4}}$$

۲

$$\textcircled{1} \sin^2 \theta + 1 - \sin^2 \theta = \frac{4}{5} \Rightarrow \sin^2 \theta - \sin^2 \theta + \frac{1}{5} \rightarrow \sin^2 \theta = \frac{5 \pm \sqrt{5}}{10}$$

$$\textcircled{2} \cos^2 \theta = \frac{5 \mp \sqrt{5}}{10} \rightarrow \cos^2 \theta + \sin^2 \theta = \frac{5 - 10\sqrt{5} + 50 + 10\sqrt{5}}{100} = \frac{55}{100} = \frac{11}{20}$$

$$\cos^2 \theta + \sin^2 \theta = \frac{50 + 10\sqrt{5} - 10\sqrt{5} + 50}{100} = \frac{100}{100} = 1$$

در هر دو حالت قبول می‌باشد.

سوال ۴

۴

$$\textcircled{1} \log \frac{\sin \alpha}{-\cos \alpha} = \log 3 \Rightarrow \frac{\sin \alpha}{-\cos \alpha} = 3 \Rightarrow \sin \alpha = -3 \cos \alpha$$

$$\textcircled{2} 9 \cos^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \cos^2 \alpha = \pm \frac{\sqrt{10}}{10} \Rightarrow \sin \alpha = \pm \frac{3\sqrt{10}}{10}$$

۳. $\sin \alpha$ باید مثبت و $\cos \alpha$ باید منفی باشند که در کمانع کای قبول باشند.

$$\textcircled{3} \frac{3\sqrt{10} + \sqrt{10}}{10} = \frac{4\sqrt{10}}{10}$$

$$\textcircled{1} \frac{\sin n \cos n - 1}{\cos n} < 0 \Rightarrow \cos n < 0$$

$$\textcircled{2} \frac{\sin n (\cos n + 3)}{3 + \sin^2 n - 1} > 0 \Rightarrow \sin n < 0$$

۵

$$1) \frac{2\cos\alpha + 3\sin\alpha}{\sin\alpha\cos\alpha} = 0 \Rightarrow 2\cos\alpha = -3\sin\alpha \Rightarrow \frac{2}{3}\sin\alpha + \sin\alpha = 1$$

$$\begin{aligned} 2) \sin\alpha &= \frac{3}{5} \\ 3) \sin\alpha &= \frac{3}{5} \end{aligned}$$

✓

$$2) \Rightarrow -\cos\alpha = \frac{3}{5} \Rightarrow \cos\alpha = -\frac{3}{5}$$

$$3) \frac{1}{\sin\alpha\cos\alpha} = \frac{1}{\frac{3}{5} \times \frac{3}{5}} = \frac{25}{9} \rightarrow \frac{13}{9} \text{ منفی}$$

✓

$$1) \sqrt{1+\sin 90^\circ} = \sqrt{1+\cos 90^\circ} \xrightarrow{1+\cos 90^\circ = 2\cos^2 45^\circ} \frac{2\sqrt{2}\cos 45^\circ}{\sqrt{2}\cos 0^\circ + \sin 0^\circ} = \sqrt{2}$$

✓

$$*) \rightarrow a\sin\alpha + b\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}\sin(\alpha+\varphi) \quad \text{if } a\alpha = \frac{b}{a} = \sqrt{2} \Rightarrow \alpha = 45^\circ$$

$$\sin\alpha = x, \cos\alpha = y \Rightarrow x^2 + y^2 = 1 \Rightarrow (x-y)^2 + y^2 = 1$$

$$2) 9y^2 - 12y + 4 + y^2 = 1 \Rightarrow 10y^2 - 12y + 3 = 0 \Rightarrow 10y^2 - 12y = -3$$

$$3) \frac{\sin\alpha}{\cos\alpha} = \tan\alpha \Rightarrow \tan(\alpha - 12^\circ) = \frac{1}{2} \Rightarrow \alpha - 12^\circ = 26.5^\circ \Rightarrow \alpha = 38.5^\circ$$

$$1) (\cos^2\alpha - \sin^2\alpha)(\cos^2\beta - \sin^2\beta)(\cos^2\gamma - \sin^2\gamma)$$

$$2) (\cos 10^\circ)(\cos 20^\circ)(\cos 40^\circ) \times \frac{\sin 10^\circ}{\sin 10^\circ} = \frac{\sin 10^\circ \times \cos 10^\circ \times \cos 20^\circ \times \cos 40^\circ}{2\sin 10^\circ}$$

$$3) = \frac{\sin 10^\circ \times \cos 10^\circ}{2\sin 10^\circ} = \frac{\sin 20^\circ}{2} = \frac{1}{4} \cos 10^\circ$$

$$1) \sin B \times \sin C = \frac{1}{4}(\cos(B-C) - \cos(B+C)) \quad \left(\cos \frac{A}{2}\right)^2 = \frac{1}{4}(\cos A + 1)$$

$$2) A+B+C=180^\circ \rightarrow B=180^\circ-A-C \Rightarrow \frac{1}{4}(\cos(180^\circ-(C+A)) - \cos(180^\circ-A))$$

$$3) \Rightarrow \frac{1}{4}(-\cos(C+A) + \cos A) = \frac{1}{4}\cos A + \frac{1}{4} = -\frac{1}{4}\cos(C+A) + \frac{1}{4}\cos A + \frac{1}{4}$$

$$4) \Rightarrow -\frac{1}{4}\cos(C+A) = \frac{1}{4} \Rightarrow \cos(C+A) = -1 \Rightarrow C+A = 180^\circ$$

$$5) B=C=45^\circ \Rightarrow A=180^\circ-90^\circ=90^\circ$$

$$\begin{aligned} \sin A \sin B &= \frac{1}{4}(\cos(A-B) - \cos(A+B)) \\ \sin A \sin B &= \frac{1}{4}(\cos(45^\circ-45^\circ) - \cos(45^\circ+45^\circ)) \\ \cos A \cos B &= \frac{1}{4}(\cos(A+B) + \cos(A-B)) \end{aligned}$$