نام و نام خانوادگی .... امیرمحمد ابوطالبی .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....
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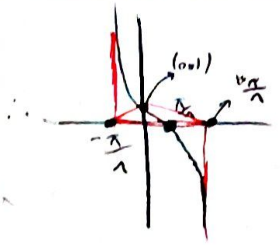

$\to \dfrac{\frac{\pi}{2}\times 1}{2} = \frac{\pi}{4}$ $\quad S_{ABC} = \frac{\pi}{4}$

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$x^2\ \ \frac{4}{9} - \frac{32}{9}\ \ \frac{-28}{9}$

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$x^2 - \sin\alpha\, x - \frac{1}{4}\cos^2\alpha = 0 \to \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} \to \dfrac{\sin\alpha \pm\sqrt{\sin^2\alpha+\cos^2\alpha}}{2} \to \dfrac{\sin\alpha+1}{2},\ \dfrac{\sin\alpha-1}{2}$

$\dfrac{\sin\alpha+1}{2} = \dfrac{2}{3} \to 3\sin\alpha+3=4 \to \sin\alpha=\frac{1}{3} \to \cos\alpha=\pm\frac{2\sqrt{2}}{3}$

$\left(\dfrac{\sin\alpha}{\cos\alpha}\right)^2 = \tan^2\alpha \to \left(\dfrac{\frac{1}{3}}{\pm\frac{2\sqrt{2}}{3}}\right)^2 \to \dfrac{1}{8}$

$1+\dfrac{1}{8} = \boxed{\dfrac{9}{8}}$

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$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \to \log\left(\dfrac{-\sin\alpha}{\cos\alpha}\right) = \log 3 \to -\tan\alpha = 3 \to \tan\alpha = -3$

$\dfrac{\sin\alpha}{\cos\alpha} = -3 \to -3\cos\alpha = \sin\alpha \to \sin^2\alpha+\cos^2\alpha=1 \to 9\cos^2\alpha+\cos^2\alpha=1 \to 10\cos^2\alpha=1 \to \cos\alpha=\pm\dfrac{1}{\sqrt{10}}$ غ ق ق

$\sin\alpha = \dfrac{+3}{\sqrt{10}}$ $\quad \sin\alpha - \cos\alpha = \dfrac{3}{\sqrt{10}} + \dfrac{1}{\sqrt{10}} = \dfrac{4}{\sqrt{10}} = \dfrac{4\sqrt{10}}{10} = \dfrac{2\sqrt{10}}{5}$

$\cos\alpha = \dfrac{-1}{\sqrt{10}}$

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$\sin^4\alpha + \cos^2\alpha = \dfrac{4}{5} \to \sin^4\alpha + 1 - \sin^2\alpha = \dfrac{4}{5} \to$ ~~...~~

$\sin^2\alpha(\sin^2\alpha - 1) = -\dfrac{1}{5} \to \sin^2\alpha \times -\cos^2\alpha = -\dfrac{1}{5} \to -(\sin^2\alpha\cos^2\alpha) = -\dfrac{1}{5}$

$\cos^4\alpha + \sin^2\alpha \to \cos^4\alpha + 1 - \cos^2\alpha = \to \cos^2\alpha(\cos^2\alpha - 1) + 1 \to \cos^2\alpha \cdot -\sin^2\alpha + 1$

$\to \underbrace{-(\sin^2\alpha\cos^2\alpha)}_{-\frac{1}{5}} + 1 \to 1 - \dfrac{1}{5} = \boxed{\dfrac{4}{5}}$

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همواره منفی

$\sin\alpha\tan\alpha - \dfrac{1}{\cos\alpha} > 0 \to \dfrac{\sin^2\alpha - 1}{\cos\alpha} > 0 \to \boxed{\cos\alpha < 0}$

همواره مثبت

$\dfrac{3\sin x + \sin x\cos x}{2(1-\cos^2 x)} < 0 \to \dfrac{\cancel{\sin x}(3+\cos x)}{2\sin^{\cancel{2}} x} < 0 \to \dfrac{3+\cos\alpha}{2\sin\alpha} < 0 \to \boxed{\sin\alpha < 0}$

ناحیه سوم → ربع سوم مثلثاتی

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$$\frac{2\cos x + 3\sin x}{\sin x\cos x} = 0 \longrightarrow 2\cos x = -3\sin x \longrightarrow \cos x = -\frac{3}{2}\sin x$$

$$\longrightarrow \sin^2 + \cos^2 = 1 \longrightarrow \sin^2 x + \frac{9}{4}\sin^2 x = 1 \rightsquigarrow \frac{13}{4}\sin^2 x = 1 \longrightarrow \sin x = \pm\sqrt{\frac{4}{13}}$$

$$\longrightarrow \underset{\text{فرض می‌کنیم}}{\sin x > 0} \longrightarrow \cos x = -\frac{3}{2}\sqrt{\frac{4}{13}}$$

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$$\tan x = \frac{\sqrt{\frac{4}{13}}}{-\frac{3}{2}\sqrt{\frac{4}{13}}} = -\frac{2}{3} \rightsquigarrow \tan x + \cot x \longrightarrow -\frac{2}{3} - \frac{3}{2} \rightsquigarrow \frac{-4}{6} - \frac{9}{6} = \frac{-13}{6}$$

$$\cot x = \frac{-\frac{3}{2}\sqrt{\frac{4}{13}}}{\sqrt{\frac{4}{13}}} = -\frac{3}{2}$$

$$\sqrt{1+\sin 50^\circ} = \sqrt{1+\cos 40^\circ} \xrightarrow[1+\cos 40^\circ = 2\cos^2 20^\circ]{\text{فرمول طلایی}} \sqrt{2\cos^2 20^\circ} = \sqrt{2}\,|\cos 20^\circ| = \sqrt{2}\cos 20^\circ$$

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$$\sin 80^\circ + \sin 40^\circ = \sin(60^\circ + 20^\circ) + \sin(60^\circ - 20^\circ) =$$

$$\sin 60^\circ\cos 20^\circ + \cancel{\sin 20^\circ\cos 60^\circ} + \sin 60^\circ\cos 20^\circ - \cancel{\sin 20^\circ\cos 60^\circ} = 2\left(\frac{1}{2}\right)\cos 20^\circ = \cos 20^\circ$$ ۷

$$\frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \boxed{\sqrt{2}}$$

$$1 - 2\sin^2 5 \longrightarrow \sin^2 5 + \cos^2 5 - 2\sin^2 5 = \cos^2 5 - \sin^2 5 = \cos 10$$

$$\frac{1-\tan^2 20}{1+\tan^2 20} = \underbrace{\cos^2 20 - \sin^2 20}_{\text{فرمول ۱۱}\ \cos 40} \sim \cos^4 10 + \sin^4 10 - 2\sin^2 10\cos^2 10 - 4\sin^2 10\cos^2 10 = \cos^4 10 + \sin^4 10 - 6\sin^2 10\cos^2 10$$ ۱

$$\underbrace{(\cos 10)(2\cos^2 10 - 1)}_{2\cos^3 10 - \cos 10}(\cos^4 10 + \sin^4 10 - 6\sin^2 10\cos^2 10) = 2\cos^7 10 + 2\cos^3 10\sin^4 10 - 12\sin^2 10\cos^5 10 - \cos^5 10 - \sin^4 10\cos 10 + 6\sin^2 10$$ ۸

$$2\cos^7(10) + 2\cos^3(10)\sin^4(10) - 12\sin^2(10)\cos^5(10) - \cos^5(10) - \sin^4(10)\cos(10) + 6\sin^2(10)\cos^3(10)$$

$$\sin\alpha + 3\cos\alpha = 2 \longrightarrow \sin\alpha = 2 - 3\cos\alpha \longrightarrow \sin^2\alpha = 4 + 9\cos^2\alpha - 12\cos\alpha$$

$$\sin^2\alpha + \cos^2\alpha = 1 \to 4 + 9\cos^2\alpha - 12\cos\alpha + \cos^2\alpha = 1 \longrightarrow 10\cos^2\alpha - 12\cos\alpha + 3 = 0$$

$$5\cos 2\alpha - 12\cos\alpha = 5(\cos^2\alpha - \sin^2\alpha) - 12\cos\alpha \longrightarrow 5\cos^2\alpha - 5\sin^2\alpha - 12\cos\alpha \longrightarrow 5\cos^2\alpha - 5(1-\cos^2\alpha) - 12\cos\alpha$$

$$5\cos^2\alpha - 5 + 5\cos^2\alpha - 12\cos\alpha \longrightarrow 10\cos^2\alpha - 12\cos\alpha - 5 \longrightarrow \underbrace{10\cos^2\alpha - 12\cos\alpha + 3}_{0} - 8 \longrightarrow \boxed{-8}$$ ۹

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$$\sin B\sin C = \cos^2\frac{A}{2} \longrightarrow \cos^2\frac{A}{2} = \frac{1+\cos A}{2} \rightsquigarrow \cos A = (\pi - (B+C)) = -\cos(B+C)$$

$$\cos A = -(\cos B\cos C - \sin B\sin C) \longrightarrow \boxed{\cos A = \sin B\sin C - \cos B\cos C}$$

$$\sin B\sin C = \frac{1 + \sin B\sin C - \cos B\cos C}{2} \longrightarrow 0 = \frac{1 + \sin B\sin C - \cos B\cos C - 2\sin B\sin C}{2}$$ ۱۰

$$1 + -(\sin B\sin C + \cos B\cos C) = 0 \rightsquigarrow \sin B\sin C + \cos B\cos C = 1 \to \cos(B-C) = 1$$

$$\cos(0^\circ) = 1 \longrightarrow B = C = 72^\circ \longrightarrow \boxed{180 - 72 - 72 = 36^\circ} \quad \boxed{A = 36^\circ}$$

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سوال ۸

$$\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ} = \frac{\cos^2 20^\circ - \sin^2 20^\circ}{\underbrace{\cos^2 20^\circ + \sin^2 20^\circ}_{1}} = \cos 40$$

$\cos(2\times 20^\circ) = \cos 40^\circ$

$1-2\sin^2 5^\circ = \cos 10^\circ$

$2\cos^2 10-1 = \cos 20^\circ$

فرمولهای

$$\underbrace{(\sin 10)\cos 10}\times\cos 20\times\cos 40 =$$

$$\frac{1}{2}\sin 20\times\cos 20\times\cos 40$$

$$\frac{1}{2}\times\frac{1}{2}\sin 40\times\cos 40 =$$

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\sin 80 = \frac{\frac{1}{8}\sin 80}{\sin 10} = \frac{1}{8}\cot 10$$

$\cot 10$