تکلیف ۱۵ ـ نوید لطیفی

(۱)

$f(0)=B \to B=\tan(\frac{\pi}{4})=1 \to B(0,1)$

$-2x+\frac{\pi}{4}=\frac{\pi}{2} \to -2x=\frac{\pi}{4} \to x=\frac{-\pi}{8} \to A(-\frac{\pi}{8},0)$

$-2x+\frac{\pi}{4}=-\frac{\pi}{2} \to x=\frac{3\pi}{8} \to B(\frac{3\pi}{8},0)$

$S=\frac{AC \times B}{2}=1\times(\frac{4\pi}{8})\times\frac{1}{2}=\frac{2\pi}{8}=\frac{\pi}{4}$

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(۲)

$\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=0 \to \frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{4}{9}-\frac{1}{4}=$

$3\sin^2\alpha-8\sin\alpha+\frac{16}{3}-\frac{9}{3}$

$\sin^2\alpha-8\sin\alpha+\frac{7}{3}\times 3=(\sin\alpha-1)(\sin\alpha-7)$

$\sin\alpha=\frac{1}{3}$ یا $\sin\alpha=\frac{7}{3}$ ❌

$\frac{9}{\sin^2\alpha}=1+\cot^2\alpha \to \cot^2\alpha=8 \to \tan^2\alpha=\frac{1}{8}$

$+\frac{1}{8}=\frac{9}{8}$

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(۳)

$\log^{-\tan\alpha}=\log^{3} \to \tan\alpha=-3$

$\sin\alpha>0,\ \cos\alpha<0 \to$ ربع ②

$\sin 2\alpha=\frac{3}{\sqrt{10}}\times\frac{1}{\sqrt{10}}=\frac{3}{10}$

$\sin\alpha-\cos\alpha=A$

$A^2=1-\sin 2\alpha$

$\frac{\pi}{2}<\alpha<\pi$

$\pi<2\alpha<2\pi \to \sin 2\alpha<0$

$1-(-\frac{3}{10})=\frac{13}{10} \to \sin\alpha-\cos\alpha=\sqrt{\frac{13}{10}}$

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(۴)

$(1-\cos^2\alpha)^2+\cos^2\alpha=\frac{4}{5} \to \cos^4\alpha-2\cos^2\alpha+\cos^2\alpha+1=\frac{4}{5} \to \cos^4\alpha-\cos^2\alpha=-\frac{1}{5}$

$\cos^4\alpha+\sin^2\alpha=\underbrace{\cos^4\alpha-\cos^2\alpha}_{-\frac{1}{5}}+1 \to \frac{4}{5}$

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(۵)

$\frac{3\sin x+\sin x\cos x}{2\sin^3 x}<0 \to \frac{\sin x(3+\cos x)}{2\times\sin x\times\sin x}<0 \quad \frac{\overbrace{3+\cos x}^{\text{همواره} >0}}{2\sin x}<0 \to \sin x<0$

$\frac{\sin^2 x}{\cos x}-\frac{1}{\cos x}=\frac{-\cos^2 x}{\cancel{\cos x}}>0 \to \cos x<0 \to$ ربع سوم

(۶)

$$\frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \qquad 2\cos x = -3\sin x \to \tan x = -\frac{2}{3}$$

$$\begin{matrix}\sin x \neq 0 \\ \cos x \neq 0\end{matrix} \to x \neq \frac{k\pi}{2} \qquad -\frac{2}{3} + \left(-\frac{3}{2}\right) = \frac{-4-9}{6} = -\frac{13}{6}$$

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(۷)

$$\frac{\sqrt{1+\sin 50}}{\sin 50 + \sin 10} = \frac{\sqrt{(\sin 25 + \cos 25)^2}}{\sin 50 + \sin 10} = \frac{\sin 25 + \cos 25}{\sin 50 + \sin 10} = \frac{\sqrt{2}\sin(\overset{45^\circ}{\cancel{\tfrac{\pi}{4}}} + 25^\circ)}{\sin 50 + \sin 10} = \frac{\sqrt{2}\sin 70}{\sin 50 + \sin 10}$$

$$\frac{\sqrt{2}\cos 20}{\sin(20+30) + \sin(30-20)} = \frac{\sqrt{2}\cos 20}{\sin 30 \cancel{\cos 20} \times 2} = \frac{\sqrt{2}}{2} \times \frac{1}{\sin 30} = \sqrt{2}$$

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(۸)

$$\underbrace{(1-2\sin^2 5^\circ)}_{\cos 10}\underbrace{(2\cos^2 10 - 1)}_{\cos 20}\underbrace{\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)}_{\cos 40} = \cos 10 \cos 20 \cos 40 \times \frac{\sin 10}{\sin 10} = \frac{1}{8}\frac{\sin 80}{\cos 80} = \frac{1}{8}\tan 80$$

$$= \frac{1}{8}\cot 10^\circ$$

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(۹)

$$5(2\cos^2\alpha - 1) - 12\cos\alpha = \boxed{\overset{3}{10\cos^2\alpha - 12\cos\alpha}} - 5 \to 3 - 5 = -2$$

$$\sin\alpha = 2 - 3\cos\alpha \xrightarrow{\text{توان ۲}} \underset{1-\cos^2\alpha}{\sin^2\alpha} = 9\cos^2\alpha - 12\cos\alpha + 4 \Rightarrow 10\cos^2\alpha - 12\cos\alpha - 3 = 0$$

$$10\cos^2\alpha - 12\cos\alpha = 3$$

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(۱۰)

$$\sin B \sin C = \frac{1+\cos A}{2} \to 2\sin B \sin C - 1 = \cos A$$

$$A+B+C = \pi \to B+C = \pi - A \to \cos(B+C) = \cos(\pi - A) = -\cos A \to \cos A = -\cos(B+C)$$

$$2\sin B \sin C - 1 = \sin B \sin C - \cos B \cos C \to \cos B \cos C + \sin B \sin C = 1$$

$$\cos(B-C) = 1 \to B - C = 0 \to B = C$$

$$A = 180 - 2B = 180 - 144 = 36^\circ$$

نویلیپی تکلیف ها