اسمی طالبی

۱۴,۲۵

(۱)

الف) $\cos^2 - \sin^2 = 3\sin x - \sin^2 - \cos^2 \Rightarrow 2\cos^2 = 3\sin x \Rightarrow 2(1-\sin^2 x) = 3\sin x$

$\Rightarrow -2\sin^2 x - 3\sin x + 2 = 0 \xrightarrow{\sin x = t} t^2 - 3t - 4 = 0 \to t = -\frac{1}{-2}$ ✓ , $t = \frac{4}{-2}$ غ

۲

$\to \sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6} , 2k\pi + \frac{5\pi}{6}$ ✓ ✓

ب) $2\cos^2 x - 1 + \cos x - 2 = 0 \xrightarrow{\cos x = t} 2t^2 + t - 3 = 0 \Rightarrow t^2 + t - 6 = 0 \to t = \frac{2}{2}$ ✓ , $t = -\frac{3}{2}$ غ

$\to \cos x = 1 \Rightarrow \boxed{x = 2k\pi}$ ✓

(۲)

الف) $(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 2\sin 2x \cos 2x \Rightarrow 2\sin 4x = -1$

$(\sin^2 x + \cos^2 x) \to 1$, $(\sin^2 x - \cos^2 x) \to -\cos 2x$

۱,۲۵

$\Rightarrow \sin 4x = -\frac{1}{2} \Rightarrow 4x = 2k\pi - \frac{\pi}{6} \to \boxed{x = k\pi - \frac{\pi}{12}}$ ✓

$4x = 2k\pi - \frac{5\pi}{6} \to \boxed{x = k\pi - \frac{5\pi}{12}}$ ✓

$\cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$

ب) $\cos^2 x - \sin^2 x = -2\sin x \cos x \Rightarrow \cos\frac{x}{2} = -\sin\frac{x}{2}$

$\Rightarrow \frac{x}{2} = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = 2k\pi - \frac{\pi}{2}}$ ✓

(۳)

الف) $2\cos x + 1 = \frac{1}{\cos x} \xrightarrow{\times \cos x} 2\cos^2 x + \cos x - 1 = 0 \xrightarrow{\cos x = t} 2t^2 + t - 1 = 0$

$\Rightarrow t^2 + t - 2 \to t = \frac{1}{2} \to \cos x = \frac{1}{2} = \boxed{2k\pi \pm \frac{\pi}{3}}$ ✓ $\cup$

$t = -\frac{2}{2} = -1 \to \boxed{x = 2k\pi + \pi}$ ✗

۱,۵

اگر ۱- = cos x باشد ۰ = sin x است و عبارت تعریف نشده میشود.
پس جواب غیر قابل قبول محسوب میشد

ب) $-\cos^2 x - \sin x \cos x = 0 \Rightarrow -\cos x(\cos x + \sin x) = 0$

$\to \cos x = 0 \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}}$

$\to \cos x = -\sin x \Rightarrow 2x = k\pi - \frac{\pi}{4}$ $\xrightarrow{\cup} k\pi + \frac{\pi}{2} \cup k\pi - \frac{\pi}{4}$

$x = \frac{k\pi}{2} - \frac{\pi}{8}$

(۴

الف) $\frac{\sqrt{2}}{2}(\cos x - \sin x) \times \frac{\sqrt{2}}{2}(\cos x + \sin x) = \frac{1}{4}$ $\frac{1}{2} \rightarrow \cos^2 - \sin^2 = \frac{1}{2}$

$\Rightarrow \cos 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$ ✓ (۱)

ب) $\cos 2x \cos\frac{\pi}{9} + \sin 2x \sin\frac{\pi}{9} = -\sin 2x$

۰

(۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \cos^2 x + \sin^2 x \Rightarrow \sin 4x = 0 \Rightarrow 4x = k\pi \Rightarrow x = \frac{k\pi}{4}$

1

(۱)

(۶

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \sin^2\frac{x}{2} = 1 + \cos^2\frac{x}{2} + 2\cos\frac{x}{2}$

$\Rightarrow 2\cos^2\frac{x}{2} + 2\cos\frac{x}{2}$ → $\cos\frac{x}{2} = 0 \Rightarrow x = -\pi , \pi \Rightarrow |\pi - (-\pi)| = 2\pi$ ✓ (۲)

→ $\cos\frac{x}{2} = -1 \Rightarrow x =$ در بازه جواب ندارد

(۷

$-\sin^2 x - \sin^2 x \cos^3 x = 0 \Rightarrow -\sin^2 x(1 + \cos^3 x)$ (۲)

→ $\sin x = 0 \Rightarrow x = 0 , \pi , 2\pi$

→ $\cos 3x = -1 \Rightarrow x = \frac{\pi}{3} , \pi , \frac{5\pi}{3}$

→ ۵ جواب

(۸

$\tan(\frac{\pi}{4} - x) = \tan(4x - \cancel{x}) \Rightarrow 14x = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{14} \rightarrow x = \frac{k\pi}{14} - k\pi$ (۱٫۵)

(۹

$\sqrt{(\sin x + \cos x)^2} = |\sin x + \cos| = \sin + \cos x = \sqrt{2}(\cos^2 x - \sin^2 x)$ (۰٫۷۵)

$(\cos x + \sin x)(\cos - \sin x)$

$\cos x - \sin x = \frac{1}{\sqrt{2}} \rightarrow x = \frac{\pi}{12}$

(۱۰)

الف) $\sin^2 x - \cos^2 x = 1 \Longrightarrow -\cos 2x = 1 \Longrightarrow \cos 2x = -1$

$2x = 2k\pi + \pi \longrightarrow \boxed{x = k\pi + \frac{\pi}{2}}$

۱٫۲۵

ب) $(\sin x - \cos x)(\underbrace{\sin^2 x + \cos^2 x}_{1} + \sin x \cos x) = -1$

سوال ۱۰

در معادلات به فرم $\pm\sin^n x \pm \cos^n x$ برابر ۱ یا ۱- فقط ۴ تا است نقاط اصلی!

ب ۱ حل کنی!

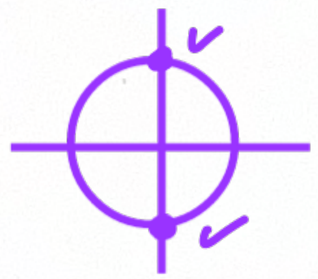
$\sin^4 x - \cos^4 x = 1$

$x = k\pi \Rightarrow 0$

$x = \frac{3\pi}{2}$

الف ۱

$\sin^4 x - \cos^4 x = 1$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

سوال ۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$

$1+\sin 2x = 2-2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0$ → $\sin 2x = -1$ ، $\sin 2x = +\frac{1}{2}$

$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0\le x\le \pi} x = \frac{3\pi}{4}$ ✓

$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0\le x\le \pi} x = \frac{\pi}{12}$ ✓ یا $\frac{5\pi}{12}$ ✗

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x=\frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد

سوال ۸

$\tan\frac{\pi}{4} = 1 \to \frac{1-\tan x}{1+\tan x} = \tan(\frac{\pi}{4}-x)$

$\tan(\frac{\pi}{4}-x) = \tan 3x \to \frac{\pi}{4} - x = k\pi + 3x$

$-4x = k\pi - \frac{\pi}{4} \xrightarrow{-k\pi = k\pi} 4x = k\pi + \frac{\pi}{4}$

$$x = \frac{k\pi}{4} + \frac{\pi}{16}$$

سوال ۲ قسمت ب

$2\cos^2 x + 2\sin x\cos x = 1$

$1+\cos$

$\rightarrow x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

سوال ۳

$$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x$$

$$\frac{\sin 4x(2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$

سوال ۴ قسمت ب

$$\cos\left(2x - \frac{\pi}{9}\right) = -\sin 2x$$

$$\cos\left(2x - \frac{\pi}{9}\right) = \cos\left(\frac{\pi}{2} + 2x\right)$$

$$2x - \frac{\pi}{9} = 2k\pi \pm \left(\frac{\pi}{2} + 2x\right)$$

حالت (+) ↗ $2k\pi + \frac{\pi}{9} + \frac{\pi}{2} = 0$ ✗

حالت (−) ↳ $2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x$

$$4x = 2k\pi - \frac{7\pi}{18} \rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{72}$$

سوال ۳ - ب

$$2(1 - \cos^2 x) - \sin x\cos x + \cos^2 x = 2$$

$$-\cos^2 x - \sin x\cos x = 0 \rightarrow \cos x(\cos x + \sin x) = 0$$

$\cos x = 0 \rightarrow x = k\pi + \frac{\pi}{2}$

$\tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

سوال ۲ الف)

$\sin^4 x - \cos^4 x = \sin 4x \longrightarrow -(\cos^2 x - \sin^2 x) = \sin 4x$

$-\cos 2x \sin 4x \longrightarrow \sin 4x + \cos 2x = 0 \longrightarrow 2\sin 2x \cos 2x + \cos 2x = 0$

$\cos 2x(2\sin 2x + 1) = 0 \longrightarrow \cos 2x = 0 \longrightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\longrightarrow \sin 2x = -\frac{1}{2} \longrightarrow x = k\pi - \frac{\pi}{12} \cup k\pi + \frac{7\pi}{12}$