۱۷٫۷۵

نام و نام خانوادگی ..... آریا طالش ..... پاسخنامه تشریحی تکلیف شماره ۱۶ ..... کلاس دهم ..... ریاضی

الف) $\cos 2x = 3\sin x - 1 \to 1+\cos 2x = 3\sin x$

$\to 2\cos^2 x = 3\sin x \to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = \frac{1}{2}$ ، $\sin x = -\frac{4}{2}$ غ ق ق

$\to x = \{2k\pi + \frac{\pi}{6} , 2k\pi + \frac{5\pi}{6}\}$

ب) $2\cos^2 x - 1 + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0 \to \cos x = 1$ ، $\cos x = -\frac{3}{2}$ غ ق ق

$\to x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin(4x) \to -\cos(2x) = \sin(4x) \to \sin(\frac{3\pi}{2} - 2x) = \sin(4x)$

$\to 4x = 2k\pi + \frac{3\pi}{2} - 2x \to 6x = 2k\pi + \frac{3\pi}{2} \to x = \frac{k\pi}{3} + \frac{\pi}{4}$

$\to 4x = 2k\pi + \pi - \frac{3\pi}{2} + 2x \to 2x = 2k\pi - \frac{\pi}{2} \to x = k\pi - \frac{\pi}{4}$

$\to x = \{\frac{k\pi}{3} + \frac{\pi}{4} , k\pi - \frac{\pi}{4}\}$

ب) $2\cos^2 x + 2\sin x\cos x = \sin^2 x + \cos^2 x \to \cos^2 x - \sin^2 x + \sin 2x = 0 \to \cos 2x = -\sin 2x$

$\to 2x = k\pi - \frac{\pi}{4} \to x = \frac{k\pi}{2} - \frac{\pi}{8}$

الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos^2 x + \cos x - 1 = 0$

$\to \cos x = -1$ — چون $\sin x = 0$ می‌شود غ ق ق

$\to \cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3}$ ۳

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2\cos^2 x + 2\sin^2 x \to \cos^2 x + \sin x\cos x = 0$

$\to \cos x(\cos x + \sin x) = 0 \to \cos x = 0 \to x = 2k\pi \pm \frac{\pi}{2}$ ، $\cos x = -\sin x \to x = k\pi - \frac{\pi}{4}$ $\to x = \{2k\pi \pm \frac{\pi}{2} , k\pi - \frac{\pi}{4}\}$

الف) $\cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4} \to \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\to \frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{4} \to \cos(2x) = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \sin(-2x) \to \sin(\frac{\pi}{2} - 2x + \frac{\pi}{9}) = \sin(\frac{11\pi}{18} - 2x) = \sin(-2x)$

$\to -2x = 2k\pi + \frac{11\pi}{18} - 2x$ غ ق ق

$\to -2x = 2k\pi + \pi - \frac{11\pi}{18} + 2x \to 4x = -2k\pi - \frac{7\pi}{18} \to x = \frac{-k\pi}{2} - \frac{7\pi}{72}$

ارفاق

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{\sin 4x}{\sin 2x} + \frac{\sin 2x}{\sin 2x} = \cos^2 x + \sin^2 x = 1$

$\to \frac{\sin 4x}{\sin 2x} + 1 = 1 \to \frac{\sin 4x}{\sin 2x} = 0 \to \sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$ ۵

$\to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$

$\to x = k\pi \pm \frac{\pi}{4}$

$$\frac{\sin(\frac{x}{2})}{1+\cos(\frac{x}{2})} = \frac{1}{\sin(\frac{x}{2})} + \frac{\cos(\frac{x}{2})}{\sin(\frac{x}{2})} \to \sin(\frac{x}{2})^2 = 1+2\cos(\frac{x}{2})+\cos(\frac{x}{2})^2$$

$$\to 1-\cos(\frac{x}{2})^2 = 1+2\cos(\frac{x}{2})+\cos(\frac{x}{2})^2 \to 2\cos(\frac{x}{2})^2+2\cos(\frac{x}{2})=0$$ ۱,۵ ۶

$$\to \cos(\frac{x}{2})^2+\cos\frac{x}{2}=0 \to \cos\frac{x}{2}=0 \to \frac{x}{2}=4k\pi+\frac{\pi}{2} \to x=2k\pi+\pi$$

$$\cos\frac{x}{2}=-1 \to \frac{x}{2}=2k\pi+\pi \to x=4k\pi+2\pi$$ توجه

$$\cos x^2 - \sin^2 x \cos 4x = \sin^2 x + \cos^2 x \to \sin^2 x + \sin^2 x \cos 4x = 0$$

$$\to \sin x^2(1+\cos 4x)=0 \to \sin x = 0 \to x = k\pi$$

$$\cos 4x = -1 \to 4x = 2k\pi+\pi \to x = \frac{2k\pi}{4}+\frac{\pi}{4}$$ ۷

۱,۵

$$\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} = \frac{\sin 3x}{\cos 3x} \to \frac{\cos-\sin x}{\cos+\sin x}\times\frac{(\cos-\sin x)}{(\cos-\sin x)} = \frac{\sin 3x}{\cos 3x}$$ ۸

$$\Rightarrow \frac{\cos x^2+\sin x^2-2\cos x\sin x}{\cos x^2-\sin x^2} = \frac{1-\sin(2x)}{\cos(2x)} = \frac{\sin 3x}{\cos 3x}$$ ۲

$$\to \cos 3x - \sin 2x \cos 3x = \sin 3x \cos 2x \to \cos 3x = \sin 5x \to \sin(\frac{\pi}{2}-3x) = \sin(5x)$$

$$\to 5x = \frac{\pi}{2}-3x+2k\pi \to 8x = 2k\pi+\frac{\pi}{2} \to x = \frac{k\pi}{4}+\frac{\pi}{16}$$

$$\to 5x = 2k\pi+\pi-\frac{\pi}{2}+3x \to 2x = 2k\pi+\frac{\pi}{2} \to x = k\pi+\frac{\pi}{4}$$

$$\sqrt{1+\sin 2x} = \sqrt{\sin x^2+\cos x^2+2\sin x\cos x} = \sqrt{(\sin x+\cos x)^2} = |\sin x+\cos x|$$ ۹

۱,۵

$$\oplus \to (\sin x+\cos x) = \sqrt{2}(\sin x+\cos x)(\cos x-\sin x) \to 1 = \sqrt{2}(\cos x-\sin x)$$

$$\to 1 = \sqrt{2}\times(-\sqrt{2}\sin(x-\frac{\pi}{4})) \to \sin(x-\frac{\pi}{4}) = \frac{-1}{2} \to x-\frac{\pi}{4} = 2k\pi - \frac{\pi}{6}$$

$$\to x-\frac{\pi}{4} = 2k\pi-\frac{5\pi}{6}$$

الف) $\sin^4 x - \cos^4 x = 1 \to -\cos(2x) = 1 \to \cos(2x) = -1 \to 2x = 2k\pi+\pi \to x = k\pi+\frac{\pi}{2}$

ب) $\to$ جواب‌ها $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$ ۱,۲۵ ۱۰

$$\to \sin^3 x - \cos^3 x = -1 \to (\sin x-\cos x)^3 + 3\sin x\cos x(\sin x-\cos x)$$

$$\ominus \to -(\sin x+\cos x) = \sqrt{2}(\cos x+\sin x)(\cos x-\sin x) \to -1 = \sqrt{2}\times(-\sqrt{2}(\sin(x+\frac{\pi}{4}))$$

$$\to \sin(x+\frac{\pi}{4}) = \frac{1}{2} \to x+\frac{\pi}{4} \to x+\frac{\pi}{4} = 2k\pi+\frac{\pi}{6}$$

$$\to x+\frac{\pi}{4} = 2k\pi+\frac{5\pi}{6}$$

$\frac{\pi}{12}, \frac{7\pi}{12}$ ۲ پاسخ

سوال ۶

$$\frac{\sin \frac{x}{2}}{1+\cos\frac{x}{2}} = \boxed{\tan \frac{x}{4}}$$

$$\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\tan\frac{x}{4}} = \boxed{\cot\frac{x}{4}}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \to \tan\frac{x}{4} = \tan\left(\frac{\pi}{2}-\frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \to \frac{x}{2} = k\pi+\frac{\pi}{2} \to x = 2k\pi+\pi$$

جواب‌ها: $k=0 \to x=\pi$

$k=-1 \to x=-\pi$ → اختلاف $= |\pi-(-\pi)| = 2\pi$

سوال ۷

$$\cos^2 x - \sin^2 x \cos 3x = 1 \xrightarrow{\cos^2 x = 1-\sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \to \sin^2 x(1+\cos 3x) = 0$$

$\sin x = 0$

$\cos 3x = -1$

$\sin x = 0 \to x = k\pi \to x = 0, \pi, 2\pi$ (۳) (۲) (۱)

$\cos 3x = -1 \to 3x = 2k\pi+\pi \to x = \frac{2k\pi+\pi}{3} \to x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (۵) (۴)

معادله ۵ جواب دارد

سوال ۱۰

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1 فقط کافی است نقاط اصلی را چک کنی!

ب) ۱

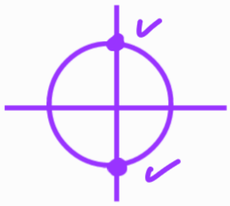
$\sin^3 x - \cos^3 x = 1$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف) ۱

$\sin^4 x - \cos^4 x = 1$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2-2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1 \text{ یا } \sin 2x = +\frac{1}{2}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$، $\cos 2x$ منفی می‌شود چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد