نام و نام خانوادگی ...... آ... طا... ...... پاسخنامه تشریحی تکلیف شماره ۱۶ ...... کلاس دوازدهم ......

الف) $\cos 2x = 3\sin x - 1 \to 1 + \cos 2x = 3\sin x$

$\to 2\cos^2 x = 3\sin x \to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = \frac{1}{2}$ ✓ ، $\sin x = -\frac{4}{2}$ غ ق ق

$\to x = \{2k\pi + \frac{\pi}{6} , 2k\pi + \frac{5\pi}{6}\}$

ب) $2\cos^2 x - 1 + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0 \to \cos x = 1$ ✓ ، $\cos x = -\frac{3}{2}$ غ ق ق

$\to x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin(4x) \to -\cos(2x) = \sin(4x) \to \sin(\frac{3\pi}{2} - 2x) = \sin(4x)$

$4x = 2k\pi + \frac{3\pi}{2} - 2x \to 6x = 2k\pi + \frac{3\pi}{2} \to x = \frac{k\pi}{3} + \frac{\pi}{4}$

$4x = 2k\pi + \pi - \frac{3\pi}{2} + 2x \to 2x = 2k\pi - \frac{\pi}{2} \to x = k\pi - \frac{\pi}{4}$

$\to x = \{\frac{k\pi}{3} + \frac{\pi}{4} , k\pi - \frac{\pi}{4}\}$

ب) $2\cos^2 x + 2\sin x\cos x = \sin^2 x + \cos^2 x \to \cos^2 x - \sin^2 x + \sin 2x = 0 \to \cos 2x = -\sin 2x$

$\to 2x = k\pi - \frac{\pi}{4} \to x = \frac{k\pi}{2} - \frac{\pi}{8}$

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الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos^2 x + \cos x - 1 = 0$

$\to \cos x = -1$ — چون $\sin x = 0$ می‌شود غ ق ق

$\to \cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2\cos^2 x + 2\sin^2 x \to \cos^2 x + \sin x\cos x = 0$

$\to \cos x(\cos x + \sin x) = 0$ ؛ $\cos x = 0 \to x = 2k\pi \pm \frac{\pi}{2}$ ؛ $\cos x = -\sin x \to x = k\pi - \frac{\pi}{4}$ $\to x = \{2k\pi \pm \frac{\pi}{2} , k\pi - \frac{\pi}{4}\}$

الف) $\cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4} \to \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\to \frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{4} \to \cos(2x) = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \sin(-2x) \to \sin(\frac{\pi}{2} - 2x + \frac{\pi}{9}) = \sin(\frac{10\pi}{18} - 2x) = \sin(-2x)$

$-2x = 2k\pi + \frac{10\pi}{18} - 2x$ غ ق ق

$-2x = 2k\pi + \pi - \frac{10\pi}{18} + 2x \to 4x = -2k\pi - \frac{8\pi}{18} \to x = \frac{-k\pi}{2} - \frac{\pi}{9}$

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{\sin 4x}{\sin 2x} + \frac{\sin 2x}{\sin 2x} = \cos^2 x + \sin^2 x = 1$

$\to \frac{\sin 4x}{\sin 2x} + 1 = 1 \to \frac{\sin 4x}{\sin 2x} = 0 \to \sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$ ۵

$2x \neq k\pi \to x \neq \frac{k\pi}{2}$

$\to x = k\pi \pm \frac{\pi}{4}$

$\frac{\sin(\frac{x}{2})}{1+\cos(\frac{x}{2})} = \frac{1}{\sin(\frac{x}{2})} + \frac{\cos(\frac{x}{2})}{\sin(\frac{x}{2})} \to \sin(\frac{x}{2})^2 = 1+2\cos(\frac{x}{2})+\cos(\frac{x}{2})^2$

$\to 1-\cos(\frac{x}{2})^2 = 1+2\cos(\frac{x}{2})+\cos(\frac{x}{2})^2 \to 2\cos(\frac{x}{2})^2+2\cos(\frac{x}{2}) = 0$ ۶

$\to \cos(\frac{x}{2})^2+\cos\frac{x}{2} = 0 \to \cos\frac{x}{2} = 0 \to \frac{x}{2} = 4k\pi+\frac{\pi}{2} \to x = 2k\pi+\pi$

$\to \cos\frac{x}{2} = -1 \to \frac{x}{2} = 2k\pi+\pi \to x = 4k\pi+2\pi$ غ ق ق

$\cos x^2 - \sin^2 x\cos 3x = \sin^2 x+\cos^2 x \to \sin^2 x+\sin^2 x\cos 3x = 0$

$\to \sin x^2(1+\cos 3x) = 0 \to \sin x = 0 \to x = k\pi$

$\to \cos 3x = -1 \to 3x = 2k\pi+\pi \to x = \frac{2k\pi}{3}+\frac{\pi}{3}$ ۷

$\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} = \frac{\sin 3x}{\cos 3x} \to \frac{\cos-\sin x}{\cos+\sin x} \times \frac{(\cos-\sin x)}{(\cos-\sin x)} = \frac{\sin 3x}{\cos 3x}$ $\cancel{\frac{1-\sin x\ \sin x}{1-\sin x^2}}$ =

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$\Rightarrow \frac{\cos x^2+\sin x^2-2\cos x\sin x}{\cos x^2-\sin x^2} = \frac{1-\sin(2x)}{\cos(2x)} = \frac{\sin 3x}{\cos 3x}$

$\to \cos 3x-\sin 2x\cos 3x = \sin 3x\cos 2x \to \cos 3x = \sin 5x \to \sin(\frac{\pi}{2}-3x) = \sin(5x)$

$\to 5x = \frac{\pi}{2}-3x+2k\pi \to 8x = 2k\pi+\frac{\pi}{2} \to x = \frac{k\pi}{4}+\frac{\pi}{8}$

$\to 5x = 2k\pi+\pi-\frac{\pi}{2}+3x \to 2x = 2k\pi+\frac{\pi}{2} \to x = k\pi+\frac{\pi}{4}$

$\sqrt{1+\sin 2x} = \sqrt{\sin x^2+\cos x^2+2\sin x\cos x} = \sqrt{(\sin x+\cos x)^2} = |\sin x+\cos x|$ ۹

$+ \to (\sin x+\cos x) = \sqrt{2}(\sin x+\cos x)(\cos x-\sin x) \to 1 = \sqrt{2}(\cos x-\sin x)$

$\to 1 = \sqrt{2}\times(-\sqrt{2}\sin(x-\frac{\pi}{4})) \to \sin(x-\frac{\pi}{4}) = \frac{-1}{2} \to x-\frac{\pi}{4} = 2k\pi - \frac{\pi}{6}$

$\to x-\frac{\pi}{4} = 2k\pi-\frac{5\pi}{6}$

الف) $\sin^4 x-\cos^4 x = 1 \to -\cos(2x) = 1 \to \cos(2x) = -1 \to 2x = 2k\pi+\pi \to x = k\pi+\frac{\pi}{2}$

$\to$ [illegible] $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$ ۱۰

ب) $\sin^3 x-\cos^3 x = -1 \to (\sin x-\cos x)^3+3\sin x\cos x(\sin x-\cos x)$

$- \to -(\sin x+\cos x) = \sqrt{2}(\cos x+\sin x)(\cos x-\sin x) \to -1 = \sqrt{2}\times(-\sqrt{2}(\sin(x+\frac{\pi}{4}))$

$\to \sin(x+\frac{\pi}{4}) = \frac{1}{2} \to x+\frac{\pi}{4} \to x+\frac{\pi}{4} = 2k\pi+\frac{\pi}{6}$

$\to x+\frac{\pi}{4} = 2k\pi+\frac{5\pi}{6}$

$\frac{\pi}{12}, \frac{7\pi}{12}$ ۲ پاسخ