۲۰ آفرین خیلی خوب نوشتی

تکلیف ۱۶

Date ______ Subject رادمهر معتمدی

۱)

$\cos(2x) = 3\sin x - 1 = 1 - 2\sin^2 x \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0 \rightarrow \sin x = \frac{1}{2} \rightarrow x = 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}$ ۲

$\sin x = -2$ ✗

$\cos(2x) + \cos x - 3 = 0 = 2\cos^2 x + \cos x - 3 = 0 = (\cos x - 1)(2\cos x + 3) = 0$

$\cos x = 1 \Rightarrow x = 2k\pi$

$\cos x = -\frac{3}{2}$ ✗

۲) الف)

$\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = -\cos 2x = \sin 4x$

$\Rightarrow 2\sin 2x \cos 2x = -\cos 2x \Rightarrow (2\sin 2x + 1)(\cos 2x) = 0 \rightarrow \sin 2x = -\frac{1}{2}$ ۲

$\cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2}$

$\rightarrow 2x = 2k\pi + \frac{7\pi}{6}, 2k\pi + \frac{11\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12}, k\pi + \frac{11\pi}{12}$

ب)

$2\cos^2 x + 2\sin x \cos x = 1 \Rightarrow \cos 2x = -\sin(2x) =$ ~~[crossed out]~~

$\rightarrow$ ~~[crossed out]~~ $\tan(2x) = -1 \rightarrow 2x = k\pi + \frac{3\pi}{4} \rightarrow x = \frac{k\pi}{2} + \frac{3\pi}{8}$

۳) الف)

$\cot(x)(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \cos x(2\cos x + 1) = 1$; $\sin x \neq 0$

$2\cos^2 x + \cos x - 1 = 0 \Rightarrow (2\cos x - 1)(\cos x + 1) = 0$ ۲

$\cos x = -1 \rightarrow \sin x = 0$ ✗

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$

ب)

$2\sin^2 x - \sin x \cos x + \cos^2 x = 2 \longrightarrow 1 - \cos 2x - \frac{1}{2}\sin 2x + \frac{1 + \cos 2x}{2} = 2$

$\rightarrow \frac{1}{2}(\sin(2x) + \cos(2x)) = -\frac{1}{2} \rightarrow \sqrt{2}\sin(2x + \frac{\pi}{4}) = -1 \rightarrow \sin(2x + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\rightarrow 2x + \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4}, 2k\pi + \frac{7\pi}{4} \rightarrow x = k\pi + \frac{3\pi}{4}, k\pi + \frac{\pi}{2}$

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۴) الف) $\cos(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4}) = \left(\frac{\sqrt{2}}{2}(\cos x - \sin x)\right)\left(\frac{\sqrt{2}}{2}(\cos x + \sin x)\right)$

$= \frac{1}{2}(\cos^2 x - \sin^2 x) = \frac{1}{4} \to \cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$ ۲

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x = \cos(2x + \frac{\pi}{2}) = \cos(-2x - \frac{\pi}{2})$

$\Rightarrow 2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \to 4x = 2k\pi - \frac{7\pi}{18} \to x = \frac{k\pi}{2} - \frac{7\pi}{72}$

۵) $\frac{\sin(4x) + \sin(2x)}{\sin(2x)} = \cos^2 x + \sin^2 x = 1 \to \sin(4x) = 0$ ۲

$\sin(2x) \neq 0 \to x \neq k\pi + \frac{\pi}{4} \to 2\sin 2x \cos 2x = 0 \to \cos 2x = 0 \to x = k\pi \pm \frac{\pi}{4}$

۶) $\frac{\sin(\frac{x}{2})}{1+\cos(\frac{x}{2})} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Longrightarrow \tan(\frac{x}{4}) = \frac{1}{\tan(\frac{x}{4})}$ ۲

$\tan(\frac{x}{4}) = \pm 1 \to \frac{x}{4} = k\pi \pm \frac{\pi}{4} \to x = 4k\pi \pm \pi \to x \in [-\pi, \frac{3\pi}{2}]$

۷) $\cos^2 x - \sin^2 x \cos 4x = 1 \Rightarrow -\sin^2 x\,(\cos 4x \overset{+1}{}) = 0$

$\sin^2 x = 0 \Rightarrow x = k\pi$ $\quad \cos 4x = -1 \Rightarrow x = \frac{2k\pi}{4} \pm \frac{\pi}{4}$ ۲

جعد $x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

۹) $\sqrt{1 + \sin 2x} = \sqrt{2}\cos(2x) \geq 0$

$\longrightarrow 1 + \sin 2x = 2\cos^2(2x) = 2 - 2\sin^2 2x \xrightarrow[t=\sin 2x]{} 2t^2 + t - 1 = (2t-1)(t+1)$

$\longrightarrow \sin 2x = -1 \to \cos(2x) = 0 ✓ \longrightarrow 2x = 2k\pi + \frac{3\pi}{2} \to x = k\pi + \frac{3\pi}{4}$ ۲

$\longrightarrow \sin(2x) = \frac{1}{2} \to \cos 2x = \pm\frac{\sqrt{3}}{2} \xrightarrow[\cos 2x \geq 0]{} \cos(2x) = \frac{\sqrt{3}}{2} \to 2x = 2k\pi + \frac{\pi}{6} \to x = k\pi + \frac{\pi}{12}$

$\Rightarrow x = \frac{3\pi}{4}, \frac{\pi}{12} \longrightarrow$ ۲ جواب

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۱) الف)

$\sin^4 x - \cos^4 x = -(\sin^2 x + \cos^2 x)(\cos^2 x - \sin^2 x) = -\cos 2x = 1$

$\cos 2x = -1 \rightarrow 2x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

ب) ۲

$\sin^3 x - \cos^3 x = -1 \rightarrow \cos^3 x - \sin^3 x = 1$

$\rightarrow (\cos x - \sin x)^3 + 3 \sin x \cos x (\cos x - \sin x) = 1$

$-\sqrt{2} \le t = \cos x - \sin x \rightarrow t^2 = 1 + 2\sin x \cos x \rightarrow \sin x \cos x = \frac{1-t^2}{2}$

$t^3 + 3\left(\frac{1-t^2}{2}\right) t = 1 \rightarrow t^3 - 3t + 2 = 0 = (t-1)^2 (t+2) \begin{cases} \rightarrow t = 1 \\ \rightarrow t = -2 \times \end{cases}$

$t = 1 \rightarrow \left.\begin{array}{l} \sin x \cos x = \frac{1-t^2}{2} = 0 \\ \cos x - \sin x = 1 \end{array}\right] \rightarrow \left.\begin{array}{l} \sin x = 0, \cos x = 1 \\ \cos x = 0, \sin x = -1 \end{array}\right\} \begin{array}{l} x = 2k\pi \\ x = 2k\pi + \frac{3\pi}{2} \end{array}$

$x = 0, \frac{3\pi}{2}, 2\pi$

۸) ۲

$\frac{1 - \tan x}{1 + \tan x} = \tan 3x \rightarrow 2 = (1 + \tan x)(1 + \tan 3x)$

$1 - \tan 3x \tan x = \tan(x) + \tan(3x) \rightarrow \frac{\tan x + \tan 3x}{1 - \tan x \tan 3x} = \tan(4x) = 1$

$4x = k\pi + \frac{\pi}{4} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$