۱- الف) $\cos 2x = 3\sin x - 1 \to \underset{1-\sin^2}{\cancel{\cos^2 x}} - \sin^2 x = 3\sin x - 1 \to 2\sin^2 + 3\sin - 2 = 0 \to \sin x = \frac{-3 \pm \overset{5}{\sqrt{9+16}}}{4}$

۲

۱۴٫۷۵

$\begin{cases} \sin x = \frac{1}{2} \\ \sin x = -2 \ \times \end{cases} \to x = 2k\pi + \frac{\pi}{6},\ 2k\pi + \frac{5\pi}{6}$ ✓

ب) $\cos 2x + \cos x - 2 = 0 \to \cos^2 x \underset{-(1-\cos^2 x)}{\cancel{- \sin^2 x}} + \cos x - 2 = 0 \to 2\cos^2 + \cos - 3 = 0 \to \begin{cases} \boxed{\cos x = 1} \\ \cos x = -\frac{3}{2} \ \times \end{cases}$

$x = 2k\pi$ ✓

۲- الف) $\underbrace{\sin^4 x - \cos^4 x}_{\sin^2 - \cos^2 = -\cos 2x} = \sin 4x \to \sin 4x + \cos 2x = 0 \to 2\sin 2x \cos 2x + \cos 2x = 0$

$\to \cos 2x(2\sin 2x + 1) = 0 \begin{cases} \cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4} \ ✓ \\ 2\sin 2x - 1 = 0 \to \sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6},\ 2k\pi + \frac{5\pi}{6} \end{cases}$

۲

$\to x = k\pi + \frac{\pi}{12},\ k\pi + \frac{5\pi}{12}$ ✓

ب) $2\cos^2 x + 2\sin x\cos x = 1 \to \cancel{2\cos^2} - \sin^2 \cancel{- \cos^2} - \sin 2x = 0 \to \cos 2x - \sin 2x = 0$

$\to \cos 2x = \sin 2x \to 2x = k\pi + \frac{\pi}{4} \to x = \frac{k\pi}{2} + \frac{\pi}{8}$ ✓

$(2\cos x - 1)(\cos x + 1)$

۳- الف) $\underset{\frac{\cos}{\sin}}{\cancel{\cot x}}(2\cos x + 1) = \frac{1}{\sin x} \to \cos x(2\cos x + 1) = 1 \to 2\cos^2 x + \cos x - 1 = 0$

۱٫۵

$\to \begin{cases} \cos x = -1 \to x = 2k\pi + \pi \ \times \\ \cos x = +\frac{1}{2} \to x = 2k\pi + \cancel{\pi} \pm \frac{\pi}{3} \end{cases} \to x = 2k\pi \pm \frac{\pi}{3}$

اگر $\cos x = -1$ باشد $\sin x = 0$ است و عبارت تعریف نشده می‌شود

پس جواب غیر قابل قبول محسوب می‌شود

کامل نی

ب) $3\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\xrightarrow{\div \sin^2} 3 - \cot x + \cot^2 x = \cancel{\frac{2}{\sin^2 x}}\ 2(1 + \cot^2) \to \cot^2 + \cot = 0 \to \cot(\cot + 1) = 0$

$\to \begin{cases} \cot x = 0 \to x = k\pi + \frac{\pi}{2} \ ✓ \\ \cot x = -1 \to x = k\pi - \frac{\pi}{4} \ ✓ \end{cases}$

۴- الف) $\cos(\frac{\pi}{4} + x)\underset{\sin(x + \frac{\pi}{4})}{\cancel{\cos(x - \frac{\pi}{4})}} = \frac{1}{4} \to \cancel{\frac{1}{2}}\sin(2x + \frac{\pi}{2}) = \underset{\frac{1}{2}}{\cancel{\frac{1}{4}}} \to \cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3}$

$\to x = k\pi \pm \frac{\pi}{6}$ ✓

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \to \cos(2x - \frac{\pi}{9}) = \sin(-2x) \to \cos(2x - \frac{\pi}{9}) = \cos(2x + \frac{\pi}{2})$

۲

$\to 2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x) \begin{cases} \cancel{2x} - \frac{\pi}{9} = 2k\pi + \cancel{2x} + \frac{\pi}{2} \to 2k\pi + \frac{\pi}{2} + \frac{\pi}{9} = 0 \ \propto \\ 2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \to 4x = 2k\pi - \frac{7\pi}{18} \to x = \frac{k\pi}{2} - \frac{7\pi}{72} \ ✓ \end{cases}$

۵- $\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{\sin 4x}{\sin 2x} \cancel{+1 - \sin^2 x = \cos^2 x} \to \frac{\sin 4x}{\sin 2x} = 0 \to \sin 4x = 0$

$\sin 4x = 0 \to 4x = k\pi \to \boxed{x = \frac{k\pi}{4}},\ \sin 2x \neq 0 \to 2x \neq k\pi \to \boxed{x \neq \frac{k\pi}{2}} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$ ✓

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$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$ $-\cot\frac{x}{2}$ $\to \sin^2\frac{x}{2} = (1+\cos\frac{x}{2})^2 \to \cancel{1}-\cos^2\frac{x}{2} = \cos^2\frac{x}{2}+\cancel{1}+2\cos\frac{x}{2}$ ۶

$\to 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \to \cos\frac{x}{2}(\cos\frac{x}{2}+1)=0$

- $\cos\frac{x}{2}=0 \to \frac{x}{2}=k\pi+\frac{\pi}{2} \to \boxed{x=2k\pi+\pi}$
- $\cos\frac{x}{2}=-1$ × ↳ نباید مخرج را صفر کند

$k=0 \to x=\pi$

$k=-1 \to x=-\pi$

$\pi-(-\pi)=2\pi$

۱٫۵

$\cancel{\cos^2x-\sin^2x\cos3x = 1}$ $\cancel{\cos^2x+\sin^2x}$ $\to -\cancel{\sin^2x}\cos3x = \cancel{\sin^2x} \to \cos3x=-1$ -۷

$\to 3x=2k\pi+\pi \to \boxed{x=\frac{2k\pi}{3}+\frac{\pi}{3}}$

۱٫۵

$\frac{1-\tan x}{1+\tan x}=\tan3x \to \frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}} = \frac{\cos x-\sin x}{\cos x+\sin x} = \frac{\sin3x}{\cos3x}$ -۸

$\to \sin3x\cos x+\sin3x\sin x = \cos3x\cos x-\cos3x\sin x \to \sin4x-\cos2x=0$

۲

$\to 2\sin2x\cos2x-\cos2x=0 \to \cos2x(2\sin2x-1)=0$

- $\cos2x=0 \to 2x=k\pi+\frac{\pi}{2} \to \boxed{x=\frac{k\pi}{2}+\frac{\pi}{4}}$ ✓
- $\sin2x=\frac{1}{2} \to 2x=2k\pi+\frac{\pi}{6},\ 2k\pi+\frac{5\pi}{6}$
  $\to \boxed{x=k\pi+\frac{\pi}{12},\ k\pi+\frac{5\pi}{12}}$ ✓

-۹

ع

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الف) $\sin^4-\cos^4=1 \to \sin^2-\cos^2=1 \to -\cos2x=1 \to \cos2x=-1 \to 2x=2k\pi+\pi \to \boxed{x=k\pi+\frac{\pi}{2}}$ -۱۰

ب) $\sin^3x-\cos^3x=-1 \to$ نقاط مرزی $\to \boxed{0, \frac{3\pi}{2}, 2\pi}$ ✓ $\boxed{x=\frac{\pi}{2}, \frac{3\pi}{2}}$ ✓

۲

سوال ۷

$$\cos^2 x - \sin^2 x \cos 3x = 1 \xrightarrow{\cos^2 x = 1 - \sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \rightarrow \sin^2 x (1 + \cos 3x) = 0 \rightarrow \begin{cases} \sin x = 0 \\ \cos 3x = -1 \end{cases}$$

$$\sin x = 0 \rightarrow x = k\pi \rightarrow x = \overset{③}{0}, \overset{②}{\pi}, \overset{①}{2\pi}$$

$$\cos 3x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi + \pi}{3} \rightarrow x = \overset{⑤}{\frac{\pi}{3}}, \pi, \overset{④}{\frac{5\pi}{3}}$$

معادله ۵ جواب دارد

سوال ۹

$$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x \xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2\cos^2 2x$$

$$1 + \sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \rightarrow \begin{cases} \sin 2x = -1 \\ \sin 2x = +\frac{1}{2} \end{cases}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \leq x \leq \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \leq x \leq \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است نمی‌تواند

* معادله ۲ جواب دارد

سوال ۴

$$\frac{\sin\frac{x}{۲}}{1+\cos\frac{x}{۲}} = \tan\frac{x}{۴}$$

$$\frac{1}{\sin\frac{x}{۲}} + \cot\frac{x}{۲} = \frac{1+\cos\frac{x}{۲}}{\sin\frac{x}{۲}} = \frac{1}{\tan\frac{x}{۴}} = \cot\frac{x}{۴}$$

$$\tan\frac{x}{۴} = \cot\frac{x}{۴} \rightarrow \tan\frac{x}{۴} = \tan\left(\frac{\pi}{۲} - \frac{x}{۴}\right)$$

$$\frac{x}{۴} = k\pi + \frac{\pi}{۲} - \frac{x}{۴} \rightarrow \frac{x}{۲} = k\pi + \frac{\pi}{۲} \longrightarrow x = ۲k\pi + \pi$$

جواب‌ها

$$k=0 \rightarrow x=\pi$$

$$k=-1 \rightarrow x=-\pi$$

$$\longrightarrow \text{اختلاف} = |\pi - (-\pi)| = ۲\pi$$