

$$\frac{\cos 2\theta}{1 - \sin^2 \theta} = \frac{2 \sin \theta}{1 - \sin^2 \theta} - 1$$

$$\sin \theta = -2 \sin \theta + \frac{1}{\sin \theta}$$

(الف)

$$\Rightarrow 2 \sin^2 \theta + 2 \sin \theta - 1 = 0 \Rightarrow \sin^2 \theta + 2 \sin \theta - \frac{1}{2} = 0$$

$$\sin \theta = \sin \frac{\pi}{2} \Rightarrow \theta = 2K\pi + \frac{\pi}{2} \quad \left(\theta = 2K\pi + (-1)^K \frac{\pi}{2} \right)$$

$$\cos 2\theta + \cos \theta - 1 = 2 \cos \theta + \cos \theta - 1 = 0 \Rightarrow \cos \theta = \frac{1}{3} \quad \text{نکته}$$

(ب)

$$\cos \theta = \cos 0 \Rightarrow \theta = (2K\pi)$$

$$\sin^2 \theta - \cos^2 \theta = \sin^2 \theta \Rightarrow \sin^2 \theta - \cos^2 \theta = 2 \sin^2 \theta \cos^2 \theta \Rightarrow -\cos^2 \theta = 2 \sin^2 \theta \cos^2 \theta$$

$$\Rightarrow 2\theta = 2K\pi - \frac{\pi}{2} \Rightarrow \theta = K\pi - \frac{\pi}{4}$$

$$2(\cos^2 \theta - 1) = -2 \sin \theta \cos \theta \Rightarrow \cos 2\theta = -\sin 2\theta$$

نکته: $\cos \theta = \sin \theta$ است و عبارت تغییر یافته شده مرشد

$$\Rightarrow \cos \theta = -\sin \theta \Rightarrow K\pi - \frac{\pi}{4} \Rightarrow \cos 2\theta = -\sin 2\theta \Rightarrow \left(\frac{K\pi - \pi}{2} \right)$$

$$\frac{\cos \theta}{\sin \theta} (2 \cos \theta + 1) = \frac{1}{\sin \theta} \Rightarrow 2 \cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = (2K\pi + \frac{\pi}{3}) \quad \text{و} \quad \theta = (2K\pi - \frac{\pi}{3})$$

$$2 \sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta - \sin \theta \cos \theta = 1$$

$$\Rightarrow -(\cos^2 \theta - \sin \theta \cos \theta) = 0 \Rightarrow -\cos \theta (\cos \theta + \sin \theta) = 0$$

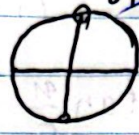
$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = (2K\pi + \frac{\pi}{2})$$

$$\Rightarrow \cos \theta = -\sin \theta \Rightarrow \theta = (K\pi - \frac{\pi}{4})$$



$$\frac{\sin^2 x + \sin^2 x}{\sin^2 x} = \sin^2 x \cdot (\cos^2 x) \Rightarrow \frac{\sin^2 x (\cos^2 x + \sin^2 x)}{\sin^2 x} = \sin^2 x + \cos^2 x \quad (1)$$

$$\Rightarrow \frac{\sin^2 x + 1}{\sin^2 x} = 1 \Rightarrow \frac{\sin^2 x}{\sin^2 x} = 0 \Rightarrow \sin^2 x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2}$$



$$\cos^2 x - \sin^2 x \cos^2 x = 1 \Rightarrow -\sin^2 x \cos^2 x = \sin^2 x$$

$$\Rightarrow -\cos^2 x = 1 \Rightarrow \cos^2 x = -1 \Rightarrow x = k\pi + \pi \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}$$

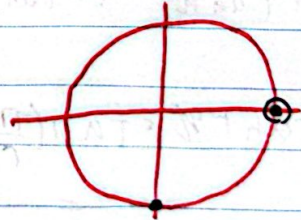
$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1$$

$$\Rightarrow \sin^2 x - \cos^2 x = 1 \Rightarrow -\cos^2 x = 1 - \sin^2 x \Rightarrow \cos^2 x = -\cos^2 x$$

$$\Rightarrow \cos^2 x = 0 \Rightarrow \cos x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2} = k\pi + \frac{\pi}{2}$$

$$\sin^2 x - \cos^2 x = -1$$

$$\sin^2 x = -1 \Rightarrow x = \left\{ 0, \frac{\pi}{2}, \pi \right\}$$



$$\frac{\sin^2 x}{1 + \cos^2 x} = \frac{\sin^2 x}{\sin^2 x} \Rightarrow \sin^2 x \cdot \frac{1 + \cos^2 x}{\sin^2 x} = 1 \Rightarrow \frac{1 + \cos^2 x}{\sin^2 x} = 1 \Rightarrow 1 + \cos^2 x = \sin^2 x$$

$$\Rightarrow 1 + \cos^2 x = \sin^2 x \Rightarrow 1 + \cos^2 x = 1 - \cos^2 x \Rightarrow 2\cos^2 x = 0 \Rightarrow \cos^2 x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2}$$

$$\Rightarrow (1 + \cos^2 x)(\cos^2 x + 1) = 0 \Rightarrow \cos^2 x = -1 \Rightarrow x = k\pi + \frac{\pi}{2}$$

$$\Rightarrow x = k\pi + \frac{\pi}{2}$$

$$\Rightarrow x = k\pi + \frac{\pi}{2}$$

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$$\Rightarrow x = \left\{ 0, -\pi, \pi, \frac{\pi}{2} \right\}$$

$$\frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1}{\sin \frac{x}{r}} + \cot \frac{x}{r} \Rightarrow \frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1 + \cos \frac{x}{r}}{\sin \frac{x}{r}} \quad (6)$$

$$\Rightarrow \tan \frac{x}{r} = \frac{1}{\tan \frac{x}{r}} \Rightarrow \tan \frac{x}{r} = \cot \frac{x}{r} \Rightarrow \frac{x}{r} = \frac{k\pi}{r} + \frac{\pi}{r} \quad (7)$$

$$\Rightarrow x \in 2k\pi + \pi \quad x \in [-\pi, \frac{3\pi}{r}] \Rightarrow \{-\pi, \pi\} \Rightarrow |\pi - (-\pi)| = 2\pi$$

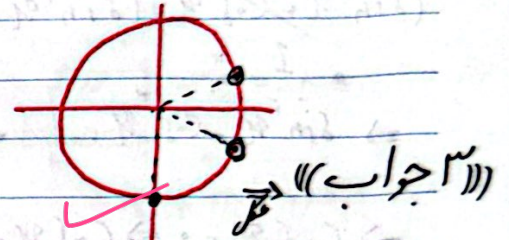
$$1 + \sin^2 x = 2 \cos^2 x \quad \text{با ازای } x = \frac{\pi}{4} \quad \text{منتهی می شود به جواب را بیاد است خود زان} \quad (9)$$

$$\sin^2 x = 2 \cos^2 x - 1 \Rightarrow \sin^2 x = 1 - 2 \sin^2 x$$

$$\Rightarrow 2 \sin^2 x + \sin^2 x - 1 = 0 \Rightarrow \sin^2 x = \frac{1}{3}$$

$$x \in [0, \pi]$$

$$2x \in [0, 2\pi]$$



$$\frac{1 - \tan x}{1 + \tan x} = \frac{\tan \frac{\pi}{4} - \tan x}{1 - \tan \frac{\pi}{4} \tan x} = \tan(\frac{\pi}{4} - x)$$

$$\Rightarrow \tan 2x = \tan(\frac{\pi}{4} - x) \Rightarrow 2x = k\pi + \frac{\pi}{4} - x \Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{12} \quad (10)$$

$$(\frac{\sqrt{2}}{r} \cos x - \frac{\sqrt{2}}{r} \sin x)(\frac{\sqrt{2}}{r} \cos x + \frac{\sqrt{2}}{r} \sin x) = \frac{1}{r} (\cos^2 x - \sin^2 x) = \frac{1}{r} \quad (11)$$

$$\Rightarrow (\cos^2 x - \sin^2 x) = \frac{1}{r} \Rightarrow 2 \cos^2 x - 1 = \frac{1}{r} \Rightarrow \cos^2 x = \frac{r+1}{2r} \Rightarrow \cos x = \frac{\sqrt{r+1}}{\sqrt{2r}} \Rightarrow x = 2k\pi \pm \frac{\pi}{2} \quad (12)$$

$$\Rightarrow \cos(2x - \frac{\pi}{4}) = -\sin 2x = \sin(-2x) = \cos(\frac{\pi}{2} + 2x)$$

$$\Rightarrow \begin{cases} 2x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{2} + 2n\pi \\ 2x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{2} + 2n\pi \end{cases}$$

$$\Rightarrow \begin{cases} 2x = 2k\pi + \frac{3\pi}{4} + 2n\pi \\ 2x = 2k\pi - \frac{\pi}{4} + 2n\pi \end{cases} \Rightarrow x = k\pi + \frac{3\pi}{8} + n\pi \quad \text{REEF GROUP}$$

سوال 9

$$\sqrt{1 + \sin 2x} = \sqrt{2} \cos x \xrightarrow{\text{توان 2}} 1 + \sin 2x = 2 \cos^2 x$$

$$1 + \sin 2x = 2 - 2 \sin^2 x \rightarrow 2 \sin^2 x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[0 \leq x \leq \pi]{k \in \mathbb{Z}} x = \frac{3\pi}{4}$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[0 \leq x \leq \pi]{k \in \mathbb{Z}} x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12}$$

چون عبارت به توان 2 رسیده است جواب را باید تقوید کردند
به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می شود پس این جواب را باید استثنای نمود

مقادیر 2 جواب دارد

سوال 7

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \xrightarrow{\cos^2 x = 1 - \sin^2 x}$$

$$1 - \sin^2 x - \sin^2 x \cos^2 x = 1 \rightarrow \sin^2 x (1 + \cos^2 x) = 0 \rightarrow \sin x = 0$$

$$\sin x = 0 \rightarrow x = k\pi \rightarrow x = 0, \pi, 2\pi$$

$$\cos^2 x = -1 \rightarrow x = 2k\pi + \pi \rightarrow x = \frac{2k\pi + \pi}{2} \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

مقادیر 5 جواب دارد