Date: ۱۴ Sub: B به نام خدا دوازده بیر ب

امیرعلی توکلی

(۱)

الف)

$\cos 2x = 3\sin x - 1$

$\Rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2$

$\Rightarrow (2\sin x - 1)(\sin x + 2) = 0 \Rightarrow \sin x = \frac{1}{2}$

$$\Rightarrow x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \pi - \frac{\pi}{6} \Rightarrow 2k\pi + \frac{5\pi}{6} \end{cases}$$

ب)

$\cos 2x + \cos x - 2 = 0 \Rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0$

$(2\cos x + 3)(\cos x - 1) \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi$

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$\sin^4 x - \cos^4 x = \sin 4x$

$\Rightarrow (\underbrace{\sin^2 x + \cos^2 x}_{1})(\sin^2 x - \cos^2 x) = \sin 4x$

$\Rightarrow -\cos 2x = 2\sin 2x \cos 2x \Rightarrow \boxed{\sin 2x = -\frac{1}{2}}$

$$\Rightarrow x = \begin{cases} 2k\pi - \frac{\pi}{6} \Rightarrow k\pi - \frac{\pi}{12} \\ 2k\pi + \pi + \frac{\pi}{6} \Rightarrow k\pi + \frac{7\pi}{12} \end{cases}$$

$+ \cos 2x = 0$

$\hookrightarrow x = k\pi + \frac{\pi}{4}$

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Date: Sub:

ب) $2\cos^2 x + 2\sin x\cos x = 1$

$\Rightarrow \cos 2x = -\sin 2x \Rightarrow 2x = 2k\pi - \frac{\pi}{4}, 2k\pi + \frac{3\pi}{4}$

$\Rightarrow x = k\pi - \frac{\pi}{8}, k\pi + \frac{3\pi}{8}$

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الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}$

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0 \Rightarrow (2\cos x - 1)(\cos x + 1) = 0$

$\cos x = -1 \Rightarrow 2k\pi + \pi, \cos x = \frac{1}{2} \Rightarrow 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \overset{\div \sin^2}{\Rightarrow}$

$\Rightarrow 2 + \cot x + \cot^2 = \frac{2}{(\sin^2 x)} = 2(1+\cot^2 x)$

$\Rightarrow 2\cot^2 x + 2 = 2 + \cot x + \cot^2 x \Rightarrow \cot^2 x + \cot x = 0$

$\Rightarrow \cot x(\cot x + 1) = 0 \rightarrow \cot x = -1 \Rightarrow x = k\pi + \frac{3\pi}{4}$

$\cot x = 0 \Rightarrow k\pi + \frac{\pi}{2}$

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الف) $\cos\left(\frac{\pi}{4}+x\right)\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{4}$

$\Rightarrow \frac{\sqrt{2}}{2}(\cos x-\sin x)\times\frac{\sqrt{2}}{2}(\cos x+\sin x) \Rightarrow \cos^2 x-\sin^2 x=\frac{1}{2}$

$\frac{1}{2}$

$\Rightarrow \cos 2x=\frac{1}{2} \Rightarrow 2x=2k\pi\pm\frac{\pi}{3} \Rightarrow x=k\pi\pm\frac{\pi}{6}$

ب) $\cos\left(2x-\frac{\pi}{9}\right)=-\sin 2x = \cos\left(2x-\frac{\pi}{9}\right)=\sin(-2x)$

$=\cos\left(2x-\frac{\pi}{9}\right)=\cos\left(\frac{\pi}{2}+2x\right) \Rightarrow 2x-\frac{\pi}{9}=k\pi\pm\left(2x+\frac{\pi}{2}\right)$

$\Rightarrow 2x-\frac{\pi}{9}=k\pi-2x-\frac{\pi}{2} \Rightarrow 4x=2k\pi-\frac{7\pi}{18} \Rightarrow x=\frac{k\pi}{2}-\frac{7\pi}{72}$

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$\frac{\sin 4x+\sin 2x}{\sin 2x}=\frac{2\sin 2x\cos 2x+\sin 2x}{\sin 2x}-\sin^2 x$

$\Rightarrow 2\cos 2x+1=\cos^2 x+\sin^2 x \Rightarrow 2(2\cos^2 x-1)+1=\cancel{\cos^2 x+\sin^2 x}\ 1$

$\Rightarrow 4\cos^2 x-2\ \cancel{+1}=\cancel{1} \Rightarrow 4\cos^2 x=2 \Rightarrow \cos x=\pm\frac{\sqrt{2}}{2}$

$\Rightarrow x=k\pi\pm\frac{\pi}{4}$

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$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \Rightarrow \frac{x}{4} = k\pi \pm \frac{\pi}{4} \Rightarrow x = 4k\pi \pm \pi$$

جواب $\Rightarrow \pi, -\pi \Rightarrow |\pi - (-\pi)| = 2\pi$

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$$\cos^2 x - (\sin^2 x \cos 3x) = 1$$

$$\cancel{\cos^2 x} - (\sin^2 x)(4\cos^3 x - 3\cos x) = 1 = \cancel{\cos^2 x} + \sin^2 x$$

$$\Rightarrow \sin^2 x(1 + \cos 3x) = 0 \Rightarrow \sin x = 0 \Rightarrow x = k\pi$$

$$\Rightarrow 0, \pi, 2\pi$$

$$\cos 3x = 2k\pi + \pi \Rightarrow \boxed{x = \frac{2k\pi}{3} + \frac{\pi}{3}} \Rightarrow \frac{\pi}{3}, \frac{5\pi}{3} \rightarrow$$ ۵ جواب

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$$\frac{1-\tan x}{1+\tan x} = \tan 3x$$

$$\Rightarrow \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \tan 3x \Rightarrow \tan\left(\frac{\pi}{4} - x\right) = \tan 3x$$

$$3x = k\pi + \frac{\pi}{4} - x \Rightarrow 4x = k\pi + \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}}$$

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$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x$$

$$1+\sin 2x = 2\cos^2 2x \Rightarrow \sin 2x = 2\cos^2 2x - 1$$

$$\Rightarrow -\sin 2x = 2\sin^2 2x - 1 \Rightarrow 2\sin^2 2x - \sin 2x - 1 = 0$$

$$\Rightarrow (2\sin^2 x - 1)(\sin 2x + 1) = 0$$

$$\begin{cases} \sin 2x = 1 \Rightarrow 2x = 2k\pi + \frac{\pi}{2} \Rightarrow x = k\pi + \frac{\pi}{4} \\ 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi + \frac{\pi}{6} \Rightarrow k\pi + \frac{\pi}{12} \end{cases}$$

زیرا کمانی دهد زیرا در $\cos 2x$ منفی

$$\Rightarrow 2k\pi + \frac{5\pi}{6} \Rightarrow k\pi + \frac{5\pi}{12}$$

$$\Rightarrow \frac{\pi}{12}, \cancel{\frac{5\pi}{12}}, \cancel{\frac{\pi}{4}}, \frac{3\pi}{4} \longrightarrow$$ ۲ جواب

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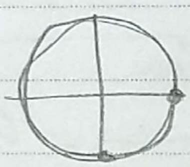
10)

الف) $\sin^2 x - \cos^2 x = 1 \Rightarrow \sin^2 x - \cos^2 x = 1$

$$\Rightarrow 1 - 2\cos^2 x = 1 \Rightarrow \cos^2 x = 0 \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}} \Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$$

ب) $\sin^3 x - \cos^3 x = -1$



$\longrightarrow$ ۳ جواب

$$\Rightarrow 0, \frac{3\pi}{2}, 2\pi$$