۱) الف) $\cos 2x = 3\sin x - 1 \to \cos^2 - \sin^2 = 3\sin x - 1 = 1 - 2\sin^2 \to 2\sin^2 + 3\sin - 2 = 0$

$(\sin + 2)(\sin - \frac{1}{2}) \to \sin = -2 \;✗ ,\; \sin = \frac{1}{2} \;✓ \to x = 2K\pi + \frac{\pi}{6} \,,\, 2K\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0$

$\cos^2 - \sin^2 + \cos - 2 = 0$

$2\cos^2 + \cos - 3 = 0 \qquad (\cos + \frac{3}{2})(\cos - 1) \qquad \cos = -\frac{3}{2} \;✗ \qquad \cos = 1 \;✓$

$x = 2K\pi$

۲) الف) $\sin^4 x - \cos^4 x = \sin 4x$

$-\cos 2x = 2\sin 2x \cos 2x$

$\cos 2x + 2\sin 2x \cos 2x = 0 \to \cos 2x(1 + 2\sin 2x) = 0$

$\cos 2x = 0 \to 2x = K\pi + \frac{\pi}{2} \to x = \frac{K\pi}{2} + \frac{\pi}{4}$

$\sin 2x = -\frac{1}{2} \to 2K\pi + \frac{11\pi}{6} \,,\, 2K\pi + \frac{7\pi}{6} = 2x$

$K\pi + \frac{11\pi}{12} \,,\, K\pi + \frac{7\pi}{12} = x$

ب) $2\cos^2 x + 2\sin x \cos x = \cancel{1}\ \sin^2 + \cos^2 \to \cos^2 - \sin^2 + 2\sin x \cos x = \cos 2x + \sin 2x = 0$

$\cos 2x = -\sin 2x \to 2x = K\pi - \frac{\pi}{4} \to x = \frac{K\pi}{2} - \frac{\pi}{8}$

$K\pi + \frac{11\pi}{12} \,,\, K\pi + \frac{7\pi}{12} \,,\, \frac{K\pi}{2} + \frac{\pi}{4}$

$x = \frac{K\pi}{2} - \frac{\pi}{8}$

۳) الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \to 2\cos^2 + \cos - 1 = 0 \quad (\frac{1}{\sin} \neq 0)$

$\cot = \frac{\cos}{\sin}$

$(\cos + 1)(\cos - \frac{1}{2}) \to \cos = \frac{1}{2} \;✓ ,\; -1 \;✗$

$\cos x = \frac{1}{2} \to x = 2K\pi \pm \frac{\pi}{3}$

$x = 2K\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 2 \xrightarrow{\div \cos^2} 2\tan^2 - \tan + 1 = \frac{2}{\cos^2} = 2 + 2\tan^2$

$\to \tan x = -1 \to x = K\pi - \frac{\pi}{4}$

$x = K\pi - \frac{\pi}{4}$

۴) الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} = \frac{1}{2}\sin(\frac{\pi}{2} + 2x) \to \sin(\frac{\pi}{2} + 2x) = \frac{1}{2}$

$\cos(x - \frac{\pi}{4}) = \sin(\frac{\pi}{4} + x)$

$\to \cos 2x = \frac{1}{2} \to 2x = 2K\pi \pm \frac{\pi}{3} \to x = K\pi \pm \frac{\pi}{6}$

$x = K\pi \pm \frac{\pi}{6}$

ب) $\cos(3x - \frac{\pi}{9}) = -\sin 2x = \cos(\frac{\pi}{2} + 2x) \Rightarrow 3x - \frac{\pi}{9} = 2K\pi \pm (2x + \frac{\pi}{2})$

$4x = 2K\pi - \frac{7\pi}{18} \to x = \frac{K\pi}{2} - \frac{7\pi}{72}$

$x = \frac{K\pi}{2} - \frac{7\pi}{72}$

۵) $\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \qquad \frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} - \sin^2$

$\sin 4x = 2\sin 2x \cos 2x$

$4\cos^2 - 1 - \sin^2 = \cos^2 \to 4\cos^2 = 2 \to \cos x = \pm\frac{\sqrt{2}}{2} \to x = K\pi \pm \frac{\pi}{4}$

$x = K\pi \pm \frac{\pi}{4}$

۶)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \qquad \frac{\cos}{\sin} \qquad \frac{1+\cos}{\sin} \qquad \cot\frac{x}{4}$$

$\tan\frac{x}{4}$

بازه $\left[-\pi, \frac{3\pi}{2}\right]$

$\tan\frac{x}{4} = \cot\frac{x}{4}$ $\qquad 2\pi$

$\tan\left(\frac{\pi}{2}-\frac{x}{4}\right) = \tan\frac{x}{4} \qquad \frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4}$

$\to \frac{x}{2} = k\pi + \frac{\pi}{2} \to x = 2k\pi + \pi \to x = -\pi, \pi \qquad |\pi-(-\pi)| = 2\pi$

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۷)

$\cos 2x - \sin^2 x \cos 3x = 1 \qquad [0, 2\pi]$

$\cos 2x - 1 = \sin^2 x \cos 3x \to \cos 3x = 1 \to 3x = 2k\pi \to x = \frac{2k\pi}{2} \to \frac{\pi}{2}, \frac{5\pi}{2}, 0$

$-\sin^2 x$

$\sin x = 0 \to x = k\pi \to \pi, 2\pi, 0$

$0, \pi, 2\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

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۸)

$\frac{1-\tan x}{1+\tan x} = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4}$

$\tan\left(\frac{\pi}{4}-x\right) \qquad x = \frac{k\pi}{4} + \frac{\pi}{16} \qquad x = \frac{k\pi}{4} + \frac{\pi}{16}$

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۹)

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \qquad [0, \pi]$

$1+\sin 2x = 2\cos^2 2x \to \sin 2x = 2\cos^2 2x - 1 = \cos 4x \Rightarrow \cos 4x = \cos\left(\frac{\pi}{2}-2x\right)$

$4x = 2k\pi \pm \left(\frac{\pi}{2} - 2x\right)$ x: $\frac{k\pi}{3} + \frac{\pi}{12}$; $k\pi - \frac{\pi}{4}$ $\to x = \frac{\pi}{12}, \frac{9\pi}{12}$

$x = \frac{\pi}{12}, \frac{9\pi}{12}$

۲ جواب

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۱۰)

الف) $\sin^4 x - \cos^4 x = 1$

$\sin^2 - \cos^2 = -\cos 2x \to \cos 2x = -1 \to 2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2} \to x = \frac{5\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}$

$\frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1 \to$ یا $\cos = +1, \sin = 0 \to 2k\pi \to 0, 2\pi$

$\sin = -1, \cos = 0 \to 2k\pi - \frac{\pi}{2} \to \frac{3\pi}{2}$

$0, \frac{3\pi}{2}, 2\pi$

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