تکلیف ۱۶ — علی درخشان فر گروه B — Note

ترتیب راه حل با (n) مشخص شده است. — Date: / /

1

الف)

① $\cos 2x = 3\sin x - 1 \xrightarrow[\cos^2 x = 1-\sin^2 x]{\cos 2x = \cos^2 x - \sin^2 x}$ ② $1 - 2\sin^2 x = 3\sin x - 1$

$\rightarrow$ ③ $2\sin^2 x + 3\sin x - 2 = 0$

- $\sin x = \frac{-3+5}{4} \Rightarrow$ ④ $\sin x = \frac{1}{2} \Rightarrow \boxed{x = 2k\pi + \frac{\pi}{6} \,,\, x = 2k\pi + \pi - \frac{\pi}{6}}$
- $\sin x = \frac{-3-5}{4}$ ✗

ب)

① $\cos 2x + \cos x - 2 = 0 \xrightarrow[\sin^2 x = 1-\cos^2 x]{\cos 2x = \cos^2 x - \sin^2 x}$ ② $2\cos^2 x + \cos x - 3 = 0$

③ $\cos x$:
- $\cos x = +1$
- $\cos x = -\frac{3}{2}$ ✗

④ جواب کلی: $2k\pi$

2

الف)

① $\sin^4 x - \cos^4 x = \sin 4x \xrightarrow[\sin 4x = 2\sin 2x \cos 2x]{\sin^4 x - \cos^4 x = -\cos 2x}$ ② $-\cos 2x = 2\sin 2x \cos 2x$

③ $\xrightarrow{\text{اگر } \cos 2x \neq 0}$ $2\sin 2x = -1 \Rightarrow \sin 2x = -\frac{1}{2} = \sin(-\frac{\pi}{6}) \Rightarrow$

- $2x = 2k\pi - \frac{\pi}{6} \rightarrow$ ⑤ $\boxed{x = k\pi - \frac{\pi}{12}}$
- $2x = 2k\pi + \pi + \frac{\pi}{6} \rightarrow$ ⑤ $x = k\pi + \frac{\pi}{2} + \frac{\pi}{12} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$

④ $\cos 2x = 0 = \cos\frac{\pi}{2} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{4}}$

ب)

① $2\cos^2 x + 2\sin x \cos x = 1 \xrightarrow[2\cos^2 x = 1+\cos 2x]{2\sin x\cos x = \sin 2x}$ ② $1 + \cos 2x + \sin 2x = 1$

③ $\cos 2x = -\sin 2x$

④ $2x$:

$2x = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

3

الف)

ساده کردن صورت سوال با ضرب کردن $\rightarrow$ ① $\frac{2\cos^2 x + \cos x}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}} \longrightarrow$ ②

②: $2\cos^2 x + \cos x - 1 = 0$

- $\cos x = -1 \longrightarrow x = 2k\pi + \pi$ ✗ $\rightarrow$ سینوس را صفر می کند!
- $\cos x = \frac{1}{2} \rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}}$

ب)

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \xrightarrow[\text{به شرط } \cos x \neq 0]{\div \cos^2 x} 2\tan^2 x - \tan x + 1 = 2(1 + \tan^2 x)$

$-\tan x = 1 \Rightarrow \boxed{x = k\pi - \frac{\pi}{4}}$

$\cos x = 0 \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}} \rightarrow$ چون اگر برابر صفر باشد معادله $\sin^2 x = 1$ بدست می آمد.

علی درخشان فر B

۱

۴ الف)

$$\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \xrightarrow{\cos(x-\frac{\pi}{4})=\cos(\frac{\pi}{4}-x)} ②$$

② : $$\cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x)=\frac{1}{4} \xrightarrow{\cos(\frac{\pi}{4}-x)=\sin(\frac{\pi}{4}+x)} \cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x)=\frac{1}{4}$$

③ $$\frac{\sin(\frac{\pi}{2}+2x)}{2}=\frac{1}{4} \longrightarrow \sin(\frac{\pi}{2}+2x)=\frac{1}{2} \longrightarrow \overset{\sin(\frac{\pi}{6})}{} \begin{cases} \frac{\pi}{2}+2x=2k\pi+\frac{\pi}{6} \to \boxed{x=k\pi-\frac{\pi}{6}} \\ 2x+\frac{\pi}{2}=2k\pi+\frac{5\pi}{6} \to \boxed{x=k\pi+\frac{\pi}{6}} \end{cases}$$

ب)

$$\cos(2x-\frac{\pi}{9})=\overset{\cos(\frac{\pi}{2}+2x)}{-\cancel{\sin 2x}} \longrightarrow \cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}+2x)$$

$$2x-\frac{\pi}{9}=2k\pi\pm(\frac{\pi}{2}+2x) \longrightarrow \cancel{2x}-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+\cancel{2x} \longrightarrow \times$$

$$\searrow 2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x \to 4x=2k\pi-\frac{7\pi}{18}$$

$$\boxed{x=\frac{k\pi}{2}-\frac{7\pi}{72}}$$

۵

$$\frac{\sin 4x}{\sin 2x}+1=\sin^2 x+\cos^2 x \Rightarrow \frac{\sin 4x}{\sin 2x}=0$$

$$\Rightarrow \begin{cases} \sin 4x=0 \Rightarrow \begin{cases} 4x=2k\pi+\pi \Rightarrow x=\frac{k\pi}{2}+\frac{\pi}{4} \\ 4x=2k\pi \to x=\frac{k\pi}{2} \end{cases} \\ \sin 2x\neq 0 \Rightarrow \begin{cases} 2x=2k\pi \to x=k\pi \text{ غق} \\ 2x=2k\pi+\pi \to x=k\pi+\frac{\pi}{2} \text{ غق} \end{cases} \end{cases} \Rightarrow \boxed{\begin{matrix} x=\frac{k\pi}{2}+\frac{\pi}{4} \\ x=\frac{k\pi}{2} \end{matrix}}$$

۶

① $$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{\cos\frac{x}{2}+1}{\sin\frac{x}{2}} \longrightarrow \underset{1-\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}}=\cos^2\frac{x}{2}+2\cos\frac{x}{2}+1$$

② $$2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0$$

③ $$\cos^2\frac{x}{2}+\cos\frac{x}{2}=0 \longrightarrow \cos\frac{x}{2}(\cos\frac{x}{2}+1)=0 \begin{cases} \cos\frac{x}{2}=\overset{\cos\frac{\pi}{2}}{\cancel{0}} \to \frac{x}{2}=2k\pi\pm\frac{\pi}{2} \\ \qquad x=4k\pi\pm\pi \\ \cos\frac{x}{2}=\overset{\cos\pi}{\cancel{-1}} \to x=4k\pi\pm2\pi \end{cases}$$

جواب ها : $\{-\pi, 0, \pi\} \Rightarrow |-\pi-\pi|=2\pi$

7

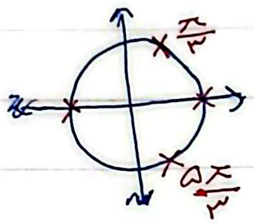
$-\sin^2 x\cos 3x = \cancel{1-\cos^2 x}$

$\sin^2 x$

$\Rightarrow \cos 3x = \overset{\cos\pi}{\cancel{-1}} \longrightarrow 3x = 2k\pi \pm \pi \rightarrow x = \frac{2}{3}k\pi \pm \frac{\pi}{3} \rightarrow \begin{cases} k=1 \Rightarrow \{\pi, \frac{\pi}{3}\} \\ k=2 \{\frac{5\pi}{3}, \pi\} \end{cases}$

$\xrightarrow{\sin^2 x=0} \sin x = 0 \Rightarrow x = 2k\pi$

$x = 2k\pi + \pi$



۵ جواب
(۲π دوبار حساب نشه)

8

① $\frac{1-\tan x}{1+\tan x} = \tan(\frac{\pi}{4}-x) = \tan 3x \rightarrow 3x = k\pi + \frac{\pi}{4} - x \rightarrow 4x = k\pi + \frac{\pi}{4}$

② $x = \frac{k\pi}{4} + \frac{\pi}{16}$

③ $\cos x \neq 0 \rightarrow x \neq 2k\pi \pm \frac{\pi}{2}$

④ $\cos 3x \neq 0 \rightarrow 3x \neq 2k\pi \pm \frac{\pi}{2} \rightarrow x \neq \frac{2k\pi}{3} \pm \frac{\pi}{6}$

⑤ $\tan x + 1 \neq 0 \rightarrow \tan x \neq -1, \tan x \neq (\frac{3\pi}{4}) \rightarrow x \neq k\pi + \frac{3\pi}{4}$

⑥ $\tan(\frac{\pi}{4}-x) = \tan 3x \Rightarrow \frac{\pi}{4} - x = k\pi + 3x \Rightarrow \boxed{x = \frac{k}{4}\pi + \frac{\pi}{16}}$

9

① $\sqrt{\sin^2 x + \cancel{2\sin x\cos x + \cos^2 x}} = \sqrt{2}\cos 2x$

$(\sin x + \cos x)^2$ $(\cos x + \sin x)(\cos x - \sin x)$

② $|\sin x + \cos x| = \sqrt{2}(\cos^2 x - \sin^2 x)$

نشد حلش کنم

اگر $1 \rightarrow \sin x + \cos x > 0 \Rightarrow \sqrt{2}(\cos x - \sin x) = 1$

10 الف)

$-\cos 2x = 1 \Rightarrow \cos 2x = \cancel{-1} \Rightarrow 2x = 2k\pi \pm \pi \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{2}}$

ب)

* برای معادلاتی که به شکل $\sin^n x \pm \cos^n x = \pm 1$ هستند برای تعیین جواب باید $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$ را امتحان کنیم

$x = 0, \frac{3}{2}\pi, 2\pi$