

۱) $1 - 2\sin^2 x + \cos^2 x - 1 \rightarrow 2\sin^2 x + \cos^2 x - 2 = 0 \xrightarrow{\sin^2 x = t} t^2 + t - 1 = 0 \Rightarrow t^2 + t - 1 = 0$
 $(t+4)(t-1) = 0 \rightarrow (t+2)(t-1) = 0 \rightarrow (\sin^2 x + 2)(2\sin^2 x - 1) = 0 \Rightarrow \begin{cases} \sin^2 x = -2 \text{ قیص} \\ \sin^2 x = \frac{1}{2} = \sin^2 \frac{\pi}{4} \rightarrow x = 2K\pi + \frac{\pi}{4} \\ x = 2K\pi + \frac{3\pi}{4} \end{cases}$
 ب) $2(\cos^2 x - 1) + \cos^2 x - 2 = 0 \rightarrow 2(\cos^2 x - 1) + \cos^2 x - 2 = 0 \rightarrow \begin{cases} \cos^2 x = 1 \rightarrow x = 2K\pi \\ \cos^2 x = -\frac{1}{2} \text{ قیص} \end{cases}$

۲) الف) $\frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{-\cos^2 x} = \sin^2 x \rightarrow -(\cos^2 x) = 2\sin^2 x \cos^2 x \rightarrow 2\sin^2 x \cos^2 x + \cos^2 x = 0$
 $\Rightarrow \cos^2 x (2\sin^2 x + 1) = 0 \rightarrow \begin{cases} \cos^2 x = 0 \Rightarrow x = K\pi + \frac{\pi}{2} \rightarrow x = \frac{K\pi}{2} + \frac{\pi}{2} \\ \sin^2 x = -\frac{1}{2} = \sin^2 -\frac{\pi}{4} \rightarrow \begin{cases} x = 2K\pi - \frac{\pi}{4} \rightarrow x = K\pi - \frac{\pi}{4} \\ x = 2K\pi + \frac{\pi}{4} \rightarrow x = K\pi + \frac{\pi}{4} \end{cases} \end{cases}$
 ب) $2(\cos^2 x - 1) + 2\sin^2 x \cos^2 x = 0 \Rightarrow \cos^2 x + \sin^2 x = 0 \rightarrow \cos^2 x = -\sin^2 x \xrightarrow{\oplus \frac{\pi}{4}} \sqrt{\frac{1}{2}}$
 $\Rightarrow x = 2K\pi - \frac{\pi}{4} \rightarrow x = K\pi - \frac{\pi}{4}$

۳) الف) $\frac{\cos^2 x}{\sin^2 x} (2\cos^2 x + 1) = \frac{1}{\sin^2 x} \xrightarrow{\sin^2 x = t} 2t^2 + t - 1 = 0 \Rightarrow \begin{cases} \cos^2 x = -1 \rightarrow \text{قیص} \\ \cos^2 x = \frac{1}{2} = \cos^2 \frac{\pi}{4} \rightarrow x = 2K\pi + \frac{\pi}{4} \end{cases}$
 ب) $2\sin^2 x - 2 - \sin^2 x (\cos^2 x + \cos^2 x) = 0$
 $2(\sin^2 x - 1) - \sin^2 x (\cos^2 x + \cos^2 x) = 0 \rightarrow -\cos^2 x - \sin^2 x (\cos^2 x + \cos^2 x) = 0 \rightarrow \cos^2 x + \sin^2 x (\cos^2 x + \cos^2 x) = 0$
 $\cos^2 x (\cos^2 x + \sin^2 x) = 0 \xrightarrow{\cos^2 x = 0 \Rightarrow K\pi + \frac{\pi}{2}} \cos^2 x = -\sin^2 x \rightarrow x = K\pi + \frac{\pi}{4}$

۴) الف) $\cos(\frac{\pi}{4} + x) \cos(\frac{\pi}{4} - x) = \frac{1}{4} \rightarrow \cos(\frac{\pi}{4} + x) \sin(\frac{\pi}{4} + x) = \frac{1}{4} \rightarrow \frac{1}{2} \sin(2x + \frac{\pi}{2}) = \frac{1}{4}$
 $\sin(2x + \frac{\pi}{2}) = \frac{1}{2} \rightarrow \cos 2x = \frac{1}{2} = \cos \frac{\pi}{3} \rightarrow \begin{cases} 2x = 2K\pi + \frac{\pi}{3} \rightarrow x = K\pi + \frac{\pi}{6} \\ 2x = 2K\pi - \frac{\pi}{3} \rightarrow x = K\pi - \frac{\pi}{6} \end{cases}$
 ب) $\cos(2x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} + 2x) \rightarrow 2x - \frac{\pi}{4} = 2K\pi + \frac{\pi}{4} + 2x \rightarrow 2K\pi + \frac{\pi}{2} = 0 \propto$
 $2x - \frac{\pi}{4} = 2K\pi - \frac{\pi}{4} - 2x \rightarrow 4x = 2K\pi + \frac{\pi}{2} \rightarrow 4x = 2K\pi + \frac{\pi}{2} \rightarrow x = \frac{K\pi}{2} + \frac{\pi}{4}$

۵) $\frac{2\sin^2 x \cos^2 x + \sin^2 x}{\sin^2 x} = 1 \rightarrow \frac{\sin^2 x (2\cos^2 x + 1)}{\sin^2 x} = 1 \xrightarrow{\sin^2 x \neq 0} 2\cos^2 x + 1 = 1 \rightarrow 2\cos^2 x = 0$
 $\Rightarrow \cos^2 x = 0 \rightarrow x = K\pi + \frac{\pi}{2} \rightarrow x = \frac{K\pi}{2} + \frac{\pi}{2}$
 $\sin^2 x \neq 0 \rightarrow x \neq K\pi \rightarrow x \neq \frac{K\pi}{2}$

$$\tan \frac{x}{\epsilon} = \frac{1}{\sin \frac{x}{\epsilon}} + \frac{\cos \frac{x}{\epsilon}}{\sin \frac{x}{\epsilon}} = \frac{1 + \cos \frac{x}{\epsilon}}{\sin \frac{x}{\epsilon}} = \cot \frac{x}{\epsilon}$$

$$\tan \frac{x}{\epsilon} = \tan\left(\frac{\pi}{\epsilon} - \frac{x}{\epsilon}\right) \rightarrow \frac{x}{\epsilon} = K\pi + \frac{\pi}{\epsilon} - \frac{x}{\epsilon} \rightarrow \frac{x}{\epsilon} = K\pi + \frac{\pi}{\epsilon} \rightarrow x = \epsilon K\pi + \pi$$

$$K = -1 \rightarrow x = -\pi \quad K = 0 \rightarrow x = \pi \quad \pi - (-\pi) = 2\pi$$

$$1 - \cos^2 x + \sin^2 x \cos^2 x = 0 \rightarrow \sin^2 x + \sin^2 x \cos^2 x = 0 \rightarrow \sin^2 x (1 + \cos^2 x) = 0 \rightarrow$$

$$\sin x = 0 \rightarrow x = K\pi \rightarrow x = 0, \pi, 2\pi$$

$$\cos^2 x = -1 \rightarrow x = \epsilon K\pi + \pi \rightarrow x = \frac{\epsilon K\pi}{\epsilon} + \frac{\pi}{\epsilon} \rightarrow x = \frac{\pi}{\epsilon}, \pi \Rightarrow \text{لا بد}$$

$$\tan\left(\frac{\pi}{\epsilon} - x\right) = \tan^2 x \rightarrow \frac{\pi}{\epsilon} - x = K\pi + x \rightarrow \epsilon x = \frac{\pi}{\epsilon} - K\pi$$

$$\Rightarrow x = \frac{\pi}{1\epsilon} - \frac{K\pi}{\epsilon}$$

$$\text{فإن } 1 + \sin^2 x = \epsilon \cos^2 x \rightarrow \sin^2 x = \epsilon \cos^2 x - 1 \rightarrow \sin^2 x = \cos^2 \epsilon x$$

$$\cos^2 \epsilon x = \cos^2\left(\frac{\pi}{\epsilon} - \epsilon x\right) \rightarrow \epsilon x = \epsilon K\pi + \frac{\pi}{\epsilon} - \epsilon x \rightarrow \epsilon x = \epsilon K\pi + \frac{\pi}{\epsilon} \rightarrow x = \frac{\epsilon K\pi}{\epsilon} + \frac{\pi}{1\epsilon}$$

$$\epsilon x = \epsilon K\pi - \frac{\pi}{\epsilon} + \epsilon x \rightarrow \epsilon x = \epsilon K\pi - \frac{\pi}{\epsilon} \rightarrow x = K\pi - \frac{\pi}{\epsilon}$$

$$\rightarrow x = \frac{\pi}{1\epsilon}, x = \frac{0\pi}{1\epsilon} \neq \frac{\pi}{\epsilon} \rightarrow \text{بالتالي فإن } x = \frac{\pi}{1\epsilon}, x = \frac{\pi}{\epsilon} \text{ (لا بد)}$$

$$\epsilon x = \frac{0\pi}{\epsilon} \rightarrow \epsilon x = \frac{\pi}{\epsilon} \rightarrow \cos \epsilon x = \cos \frac{\pi}{\epsilon} \rightarrow \text{فإن } \epsilon x = \frac{\pi}{\epsilon}$$

$$\text{الف) } \sin^2 x - \cos^2 x = 1 \rightarrow -\cos^2 x = 1 \rightarrow \cos^2 x = -1 = \cos^2 \pi \rightarrow \epsilon x = \epsilon K\pi + \pi \rightarrow x = K\pi + \frac{\pi}{\epsilon}$$

$$\Rightarrow \begin{cases} K=0 \rightarrow \frac{\pi}{\epsilon} & K=1 \rightarrow \frac{2\pi}{\epsilon} \\ K=-1 \rightarrow \frac{-\pi}{\epsilon} & K=-2 \rightarrow \frac{-2\pi}{\epsilon} \end{cases} \rightarrow \frac{\pi}{\epsilon}, \frac{2\pi}{\epsilon}$$

$$\text{ب) } \text{تقاربي المعادلتين} \rightarrow K=0 \rightarrow 0 - (1)^2 = -1 \checkmark$$

$$x = \frac{\pi}{\epsilon} \rightarrow 1^2 - 0^2 = 1 \checkmark$$

$$x = \pi \rightarrow 0 - (-1)^2 = -1 \checkmark$$

$$x = \frac{2\pi}{\epsilon} \rightarrow (1)^2 - 0^2 = 1 \checkmark$$

$$\Rightarrow x = 0, \frac{\pi}{\epsilon}, \pi$$