تکلیف ۱۶_ نوید امینی

در سوالات ~ برای بعضی جاها $\cos x$ را با $C x$ و $\sin x$ را با $S x$ نشون دادم

①

الف) $\cos 2x = 3\sin x - 1 \to 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$S^2 x + 3Sx - 4 = 0$

$(Sx+4)(Sx-1) = 0$

$Sx = \frac{1}{2}$ ✓ و $-\frac{4}{2}$ ×

$x = 2k\pi + \frac{\pi}{6}$ و $2k\pi + \frac{5\pi}{6}$

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ب) $\cos 2x + \cos x - 2 = 0 \to 2C^2x + Cx - 3$

$C^2x + Cx - 6 = 0 \quad (Cx+3)(Cx-2) = 0$

$Cx = -\frac{3}{2}$ × $\quad Cx = 1 \to x = 2k\pi$

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②

الف) $S^4 x - C^4 x = S4x \to -C2x = S4x$

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$S(2x - \frac{\pi}{2}) = S4x \to 4x = 2k\pi + 2x - \frac{\pi}{2} \to x = k\pi - \frac{\pi}{4}$

$4x = 2k\pi - 2x + \frac{\pi}{2} \to x = \frac{k\pi}{3} + \frac{\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \to 2\cos^2 x - 1 = -\sin 2x$

$\cos 2x = -\sin 2x \to \tan 2x = -1 \to 2x = k\pi - \frac{\pi}{4}$

$x = \frac{k\pi}{2} - \frac{\pi}{8}$

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③

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \to \frac{\cos x}{\cancel{\sin x}}(2\cos x + 1) = \frac{1}{\cancel{\sin x}} \to 2\cos^2 x + \cos x - 1 = 0$

$\sin x \neq 0$

$\cos^2 x + \cos x - 2 = 0$

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$(\cos x + 2)(\cos x - 1) = 0$

$\cos x = \frac{-2}{2} = -1 \quad \cos x = \frac{1}{2}$

$x = 2k\pi + \pi$

$\sin x \neq 0 \to$ ×

$x = 2k\pi \pm \frac{\pi}{3}$

ب) $\underbrace{\cos(2x - \frac{\pi}{9}) = \cos(2x + \frac{\pi}{2})}_{\text{سوال ۴ قسمت ب اشتباهی اینو نوشتم}} \to \cancel{2x} - \frac{\pi}{9} = 2k\pi + \cancel{2x} + \frac{\pi}{2}$ ×

$2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \to 4x = 2k\pi - \frac{\pi(9-2)}{18}$

$x = \frac{k\pi}{2} - \frac{7\pi}{72}$

الف) $\cos(\underbrace{\frac{\pi}{4}+x}_{\alpha})\cos(\underbrace{x-\frac{\pi}{4}}_{\beta}) = \frac{1}{4} \to \cos\left(\frac{\pi}{2}+(x-\frac{\pi}{4})\right)\cos(x-\frac{\pi}{4}) = \frac{1}{4}$

$\alpha-\beta=\frac{\pi}{2} \to \alpha=\beta+\frac{\pi}{2}$

$-\sin(x-\frac{\pi}{4})\cos(x-\frac{\pi}{4}) \to -\sin 2x = \frac{1}{2}$ ۲

$\sin 2x = -\frac{1}{2} = \sin(-\frac{\pi}{6})$

$2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$ $\quad$ $2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \xrightarrow[\cos \neq 0]{\div \cos^2 x} 2\tan^2 x - \tan x + 1 = 2(1+\tan^2 x)$

سوال تقسیم بر استثنایی
وارد کردم

$\cancel{2\tan^2 x} - \tan x + 1 = \cancel{2\tan^2 x} + 2$

$\tan x = -1 \quad x = k\pi - \frac{\pi}{4}$

① حالت $\cos x = 0 \to 2(1-\cos^2 x) = 2 \checkmark \to \cos x = 0$ جواب است $\to x = k\pi + \frac{\pi}{2}$

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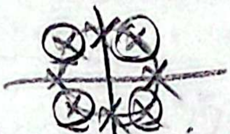
⑤

$\frac{\sin 4x + \sin^2 2x}{\sin^2 2x} - \sin^2 x = \cos^2 x$ ۲

$\sin^2 2x \neq 0$ و $\frac{\sin 4x}{\sin^2 2x} + (1-\sin^2 x) = \cancel{\cos^2 x} \to \sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$

$\sin 2x \neq 0 \to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$



جواب $\to \frac{k\pi}{2} + \frac{\pi}{4}$

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⑥

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \xrightarrow{\frac{x}{2}=\alpha} \frac{\sin\alpha}{1+\cos\alpha} = \frac{1}{\sin\alpha} + \cot\alpha$ ۱/۵

$\boxed{-\pi \le x \le 2\pi \quad -\frac{\pi}{2} \le \alpha \le \frac{2\pi}{2}}$

$\frac{\sin\alpha}{1+\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$

$\sin^2\alpha = \cos^2\alpha + 1 + 2\cos\alpha$

$\cancel{1} - \cos^2\alpha = \cos^2\alpha + \cancel{1} + 2\cos\alpha$

$2\cos\alpha(\cos\alpha + 1) = 0$

$\cos\alpha = 0 \to \alpha = k\pi + \frac{\pi}{2}$ $\quad$ $\cos\alpha = -1 \quad \alpha = 2k\pi + \pi$

α ها $= -\frac{\pi}{2}, \pi \xrightarrow{\alpha = \frac{x}{2}}$ x ها: $-\pi, 2\pi$

$2\pi - (-\pi) = 3\pi$

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⑦

$\cancel{1} - \sin^2 x - \sin^2 x\cos 3x = \cancel{1}$ ۲

$\sin^2 x(1+\cos 3x) = 0$

$\begin{cases} \sin x = 0 \to x = k\pi \to 0, \pi, 2\pi \\ \cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2k\pi}{3} + \frac{\pi}{3}, \ (2k+1)\frac{\pi}{3} \end{cases}$

$\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

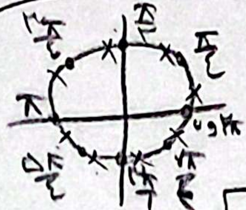
جواب $= \{0, 2\pi, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$ $\quad$ ۵ جواب

نوید امینی

۸

$$\frac{1-\tan x}{1+\tan x} = \frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} = \tan\left(\frac{\pi}{4}-x\right) = \tan 3x$$

$3x = k\pi + \frac{\pi}{4} - x \to x = \frac{k\pi}{4} + \frac{\pi}{16}$



$x = (4k+1)\frac{\pi}{16} \to \left\{\frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16}\right\}$

$\tan x \neq -1 \to x \neq k\pi - \frac{\pi}{4}$

$\cos 3x \neq 0 \to 3x \neq k\pi + \frac{\pi}{2} \to x \neq \frac{k\pi}{3} + \frac{\pi}{6}$

$\cos 2x \neq 0 \to x \neq k\pi + \frac{\pi}{4}$

$\to x \neq (2k+1)\frac{\pi}{4}$

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$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \to \cos 2x \geq 0 \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$ ۱

$\sin 2x = \cos 4x = 1 - 2\sin^2 2x$

$\to 2\sin^2 2x + \sin 2x - 1 = 0 \quad \sin^2 2x + \sin 2x - \frac{1}{2} = 0 \to (\sin 2x + 2)(\sin 2x - 1) = 0$

(I) $\sin 2x = -\frac{2}{2}$ $\quad \sin 2x = \frac{1}{2}$ (II)

$\sin 2x = -1 \to x = \frac{3\pi}{4}$

$\sin 2x = \frac{1}{2} \to x = \frac{5\pi}{12}, \frac{\pi}{12}$

(I) $\sin 2x = -1 \to 2k\pi - \frac{\pi}{2} = x$

(II) $\sin 2x = \frac{1}{2} \to x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \frac{5\pi}{6}$

ب $x \in \boxed{\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}} \xrightarrow{\cos 2x \geq 0} \cos\frac{\pi}{3} > 0 \quad \cos\frac{5\pi}{3} > 0$

دقت کن!

۱۰

الف) $\sin^4 x - \cos^4 x = 1 \to -\cos 2x = 1 \to \cos 2x = -1 \to 2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2}$

جواب: $\frac{\pi}{2}$ و $\frac{3\pi}{2}$

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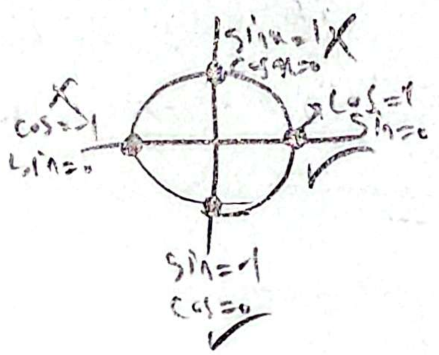
ب) $\sin^3 x - \cos^4 x = -1$



$\to x \in \left\{0, \frac{3\pi}{2}, 2\pi\right\}$

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۱۴ نوید امینی – تکلیف

سوال ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \boxed{\tan\frac{x}{4}}$$

$$\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\tan\frac{x}{4}} = \boxed{\cot\frac{x}{4}}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \rightarrow \tan\frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$$

جواب ها: $k=0 \rightarrow x=\pi$

$k=-1 \rightarrow x=-\pi$ $\rightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1 \ ,\ \sin 2x = +\frac{1}{2}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0\le x\le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0\le x\le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد