تکلیف ۱۶_ نوید امینی

در سوالات برای بعضی جاها $\cos x$ را با c و $\sin x$ را با s نشون دادم

①

الف) $\cos 2x = 3\sin x - 1 \rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$s^2x + 3sx - 4 = 0$

$(sx+4)(sx-1) = 0$

$sx = \frac{1}{2}$ ✓ و $-\frac{4}{2}$ ✗

$x = 2k\pi + \frac{\pi}{6}$ و $2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2c^2x + cx - 3$

$c^2x + cx - 6 = 0 \quad (cx+3)(cx-2) = 0$

$cx = -\frac{3}{2}$ ✗ $\quad cx = 1 \rightarrow x = 2k\pi$

---

②

الف) $s^4x - c^4x = s4x \rightarrow -c2x = s4x$

$s(2x - \frac{\pi}{2}) = s4x \rightarrow 4x = 2k\pi + 2x - \frac{\pi}{2} \rightarrow x = k\pi - \frac{\pi}{4}$

$4x = 2k\pi - 2x + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{3} + \frac{\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \rightarrow 2\cos^2 x - 1 = -\sin 2x$

$\cos 2x = -\sin 2x \rightarrow \tan 2x = -1 \rightarrow 2x = k\pi - \frac{\pi}{4}$

$x = \frac{k\pi}{2} - \frac{\pi}{8}$

---

③

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \rightarrow \frac{\cos x}{\cancel{\sin x}}(2\cos x + 1) = \frac{1}{\cancel{\sin x}} \rightarrow 2\cos^2 x + \cos x - 1 = 0$

$\sin x \neq 0$

$c^2x + \cos x - 2 = 0$

$(\cos x + 2)(\cos x - 1) = 0$

$\cos x = \frac{-2}{2} = -1 \quad \cos x = \frac{1}{2}$

$x = 2k\pi + \pi$

$\sin x \neq 0 \rightarrow$ ✗

$x = 2k\pi \pm \frac{\pi}{3}$

ب) $\cos(2x - \frac{\pi}{9}) = \cos(2x + \frac{\pi}{2}) \rightarrow \cancel{2x} - \frac{\pi}{9} = 2k\pi + \cancel{2x} + \frac{\pi}{2}$ ✗

$2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \rightarrow 4x = 2k\pi - \frac{\pi(9-2)}{18}$

$x = \frac{k\pi}{2} - \frac{7\pi}{72}$

سوال ۴ قسمت ب اشتباهی اینو نوشتم

۴

الف) $\underbrace{\cos\left(\frac{\pi}{4}+x\right)}_{\alpha}\ \underbrace{\cos\left(x-\frac{\pi}{4}\right)}_{\beta} = \frac{1}{4} \rightarrow \cos\left(\frac{\pi}{2}+\left(x-\frac{\pi}{4}\right)\right)\cos\left(x-\frac{\pi}{4}\right) = \frac{1}{4}$

$\alpha - \beta = \frac{\pi}{2} \rightarrow \alpha = \beta + \frac{\pi}{2}$

$-\sin\left(x-\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right) \rightarrow -\sin 2x = \frac{1}{2}$

$\sin 2x = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$

$2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$ $\qquad 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 3 \xrightarrow[\cos \neq 0]{\div \cos^2 x} 2\tan^2 x - \tan x + 1 = 3(1+\tan^2 x)$

سوال ۳ قسمت ب اشتباهی وارد کردم

$\cancel{2\tan^2 x} - \tan x + 1 = \cancel{2\tan^2 x} + 3$

$\tan = -1 \quad x = k\pi - \frac{\pi}{4}$

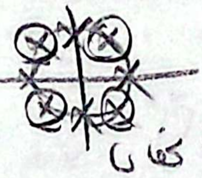
① حالت $\cos x = 0 \rightarrow 3(1-\cos^2 x) = 3$ ✓ $\rightarrow \cos x = 0 \rightarrow$ جواب است $\rightarrow x = k\pi + \frac{\pi}{2}$

۵

$\frac{\sin 4x + \sin^2 2x}{\sin^2 2x} - \sin^2 2x = \cos^2 2x$

$\sin^2 2x \neq 0$ و $\frac{\sin 4x}{\sin^2 2x} + \cancel{(1-\sin^2 2x)} = \cancel{\cos^2 2x} \rightarrow \sin 4x = 0 \rightarrow 4x = k\pi \rightarrow x = \frac{k\pi}{4}$

$\sin 2x \neq 0 \rightarrow 2x = k\pi \rightarrow x \neq \frac{k\pi}{2}$



نقاط $\rightarrow$ جواب $\rightarrow \frac{k\pi}{2} + \frac{\pi}{4}$

۶

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \xrightarrow{\frac{x}{2}=\alpha} \frac{\sin\alpha}{1+\cos\alpha} = \frac{1}{\sin\alpha} + \cot\alpha$

$-\pi \leq x \leq 3\pi$

$-\frac{\pi}{2} \leq \alpha \leq \frac{3\pi}{2}$

$\frac{\sin\alpha}{1+\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$

$2\sin^2\alpha = \cos^2\alpha + 1 + 2\cos\alpha$

$\cancel{2} - 2\cos^2\alpha = \cos^2\alpha + \cancel{1} + 2\cos\alpha$

$2\cos\alpha(\cos\alpha + 1) = 0$

$\cos\alpha = 0 \qquad \cos\alpha = -1$

$\alpha = k\pi + \frac{\pi}{2} \qquad \alpha = 2k\pi + \pi$

α ها $= -\frac{\pi}{2}, \pi \xrightarrow{\alpha = \frac{x}{2}}$ x ها: $-\pi, 2\pi$

$2\pi - (-\pi) = 3\pi$

۷

$\cancel{1} - \sin^2 x - \sin^2 x\cos^3 x = \cancel{1}$

$\sin^2 x(1+\cos^3 x) = 0$

$\sin x = 0 \rightarrow x = k\pi \rightarrow 0, \pi, 2\pi$

$\cos^3 x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3} \quad (2k+1)\frac{\pi}{3}$

$\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

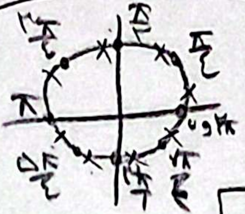
جواب $\rightarrow \left\{0, 2\pi, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ صحیح

نوید امینی

۸

$$\frac{1-\tan x}{1+\tan x} = \frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} = \tan\left(\frac{\pi}{4}-x\right) = \tan 3x$$

$3x = k\pi + \frac{\pi}{4} - x \to x = \frac{k\pi}{4} + \frac{\pi}{16}$



$x = (4k+1)\frac{\pi}{16} \to \left\{\frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16}\right\}$

$\tan x \neq -1 \to x \neq k\pi - \frac{\pi}{4}$

$\cos 3x \neq 0 \to 3x \neq k\pi + \frac{\pi}{2} \to x \neq \frac{k\pi}{3} + \frac{\pi}{6}$

$\to x \neq (2k+1)\frac{\pi}{4}$

$\cos x \neq 0 \to x \neq k\pi + \frac{\pi}{2}$

۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 3x \to \cos 3x \geq 0 \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 3x$

$\sin 2x = \cos 6x = 1 - 2\sin^2 3x$

$\to 2\sin^2 3x + \sin 2x - 1 = 0 \quad \sin^2 x + \sin x - \frac{1}{2} = 0 \to (\sin x + 2)(\sin x - 1) = 0$

I $\sin x = -\frac{\pi}{2}$ $\quad \sin x = \frac{1}{2}$ II

I $\sin x = -1 \to 2k\pi - \frac{\pi}{2} = x$

II $\sin x = \frac{1}{2} \to x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \frac{5\pi}{6}$

بازه $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} \xrightarrow{\cos 3x \geq 0} \cos\frac{\pi}{2} \geq 0 \quad \cos\frac{5\pi}{2} \geq 0$

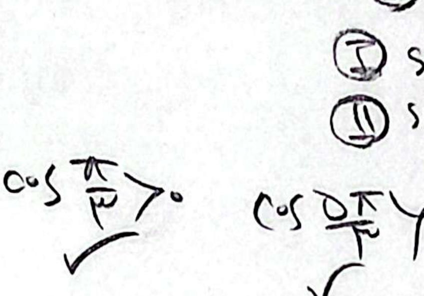
۱۰

الف) $\sin^4 x - \cos^4 x = 1 \to -\cos 2x = 1 \to \cos 2x = -1 \to 2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2}$

بازه $\frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$



$\to x_{\text{بازه}} = 0, \frac{3\pi}{2}, 2\pi$

---

۱۶ تکلیف — نوید امینی