۱۹ آفرین

Subject: Date: / /

۸۷، شماره تکلیف   سجاد سبحانی

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① $8\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow$

$2\cos x = 1 \Rightarrow \cos x = \frac{1}{2} \Rightarrow$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

← دو جواب دارد

② $5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2 \Rightarrow$

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$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \Rightarrow$

$(\cos x+1)(8\cos^2 x - 13\cos x + 7) = 0 \Rightarrow \cos x = -1 \rightarrow$

$\Delta < 0$ ← ریشه ندارد $\boxed{x = \pi, -\pi}$

دو جواب دارد

③ $2\sin x(1-2\sin^2 x) + \sin x = 1 \Rightarrow 4\sin^3 x - 3\sin x + 1 = 0 \Rightarrow$

$(\sin x+1)(\underbrace{4\sin^2 x - 4\sin x + 1}_{(2\sin x - 1)^2}) = 0 \rightarrow \begin{cases} \sin x = -1 \rightarrow x = \frac{3\pi}{2} \\ \sin x = \frac{1}{2} \begin{cases} x = \frac{\pi}{6} \\ x = \frac{5\pi}{6} \end{cases} \end{cases}$

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← مجموع جواب ها $= \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \boxed{\frac{5\pi}{2}}$

④ $\frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0 \Rightarrow \cos 2x = 2\sin 2x\cos 2x \Rightarrow$

$\cos 2x(2\sin 2x - 1) = 0 \Rightarrow$ ۲

~~$2x = 2k\pi \rightarrow x = k\pi \rightarrow x = 0, \pi$~~

$2x = 2k\pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12} \rightarrow \boxed{x = \frac{\pi}{12}}$ $\alpha = \frac{\pi}{3} \rightarrow$

$2x = 2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{5\pi}{12} \rightarrow \boxed{x = \frac{5\pi}{12}}$ $\tan 2\alpha = -\sqrt{3}$

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$\cos(2x+\frac{\pi}{2}) = \cos 2(x+\frac{\pi}{4}) = 2\cos^2(x+\frac{\pi}{4}) - 1 = \frac{2}{4} - 1 = -\frac{1}{3}$ ۱٫۵ ۵

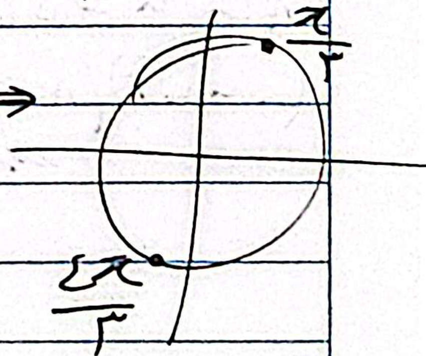
$\sin 2x = -\frac{1}{3} \Rightarrow x(\cos x - \sin x) = 2\sqrt{6}(-\frac{1}{2}) = \sqrt{6} \Rightarrow$

$x(\cos x - \sin x) = 0 \Rightarrow \boxed{x = 0}$

$\cos(\frac{\pi}{2} - x) = \sin(x + \frac{\pi}{4}) \Rightarrow$ ۶

$\sin^2(x + \frac{\pi}{4}) = 1 \Rightarrow x + \frac{\pi}{4} = k\pi \pm \frac{\pi}{2} \rightarrow$

$x + \frac{\pi}{4} = k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = k\pi + \frac{\pi}{4}} \Rightarrow$



$(\frac{4\pi}{3}, \frac{\pi}{3})$ تعداد کل جواب ها [illegible] دوتاست

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$2\sin(x + \frac{\pi}{3}) = \sqrt{2} \rightarrow \sin(x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2} \rightarrow$

$x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \Rightarrow x = 2k\pi - \frac{\pi}{12} \rightarrow -\frac{\pi}{12}, \frac{23\pi}{12}$ ۲

$x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \Rightarrow x = 2k\pi + \frac{5\pi}{12} \rightarrow \frac{5\pi}{12}$

مجموع جواب ها: $-\frac{\pi}{12} + \frac{23\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$

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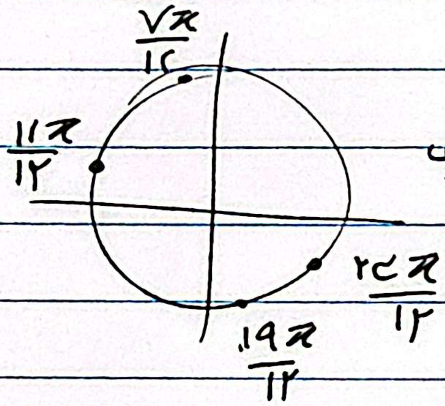
$-4\sin x\cos x = 1 \Rightarrow \sin 2x = -\frac{1}{2} \rightarrow$

$-2\sin 2x$

$2x = 2k\pi + (-\frac{\pi}{6}) \Rightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \frac{7\pi}{6} \Rightarrow x = k\pi + \frac{7\pi}{12}$



مجموع جواب ها $\Rightarrow \frac{60\pi}{12} = 5\pi$

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$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = 8(\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha) = \frac{1}{8} \Rightarrow$

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$\dfrac{\frac{1}{8}\sin^2 8\alpha}{\sin^2\alpha} = \frac{1}{8} \Rightarrow |\sin 8\alpha| = |\sin\alpha| \rightarrow 8\alpha = k\pi \pm \alpha \Rightarrow$

$\hookrightarrow \alpha = \pi \rightarrow \sin\alpha = 0$ × غ ق ق

$\alpha = \frac{k\pi}{7}$, $\alpha = \frac{k\pi}{9}$ $\Rightarrow [\pi]$ بزرگترین تعداد در مجموعه A خود π است X

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$\cos 3x = \sin x \rightarrow 3x = k\pi + \frac{\pi}{2} - x \Rightarrow$

$\cos(\frac{\pi}{2} - x)$

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$x = \frac{k\pi}{4} + \frac{\pi}{8} = \left[\frac{2k\pi + \pi}{8}\right] \rightarrow x = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

مجموع جواب ها $= \frac{48\pi}{8} = [6\pi]$

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$$\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{\sqrt{3}}{3}\longrightarrow \cos x-\sin x=\frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x-\sin x=\frac{\sqrt{2}}{\sqrt{3}}\xrightarrow{\text{گویا}}\frac{\sqrt{6}}{3}\longrightarrow\boxed{\cos x-\sin x=\frac{\sqrt{6}}{3}}$$

$$(\cos x-\sin x)^2=1-\sin 2x=\frac{6}{9}\longrightarrow\boxed{\sin 2x=\frac{1}{3}}$$

$$m(\cos x-\sin x)-3\sqrt{6}(\sin 2x)=\sqrt{6}\longrightarrow\frac{\sqrt{6}}{3}m-3\sqrt{6}\left(\frac{1}{3}\right)=\sqrt{6}$$

$$\frac{\sqrt{6}}{3}m-\sqrt{6}=\sqrt{6}\longrightarrow\frac{m\sqrt{6}}{3}=2\sqrt{6}\longrightarrow\boxed{m=6}$$

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$$1+\cos 2* = 2\cos^2 *$$

$$\begin{cases}1+\cos 2\alpha=2\cos^2\alpha\\ 1+\cos 4\alpha=2\cos^2 2\alpha\\ 1+\cos 8\alpha=2\cos^2 4\alpha\end{cases}\longrightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{64}\xrightarrow{\sqrt{\ }}\cos\alpha\cos 2\alpha\cos 4\alpha=\frac{\pm 1}{8}\xrightarrow[\sin\alpha\neq 0]{\times\sin\alpha\ *}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha=\pm\frac{\sin\alpha}{8}\longrightarrow\frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha=\pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha}=\frac{\pm\sin\alpha}{8}\longrightarrow\frac{1}{\cancel{8}}\sin 8\alpha=\pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha=\pm\sin\alpha\longrightarrow\begin{cases}8\alpha=2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha\\ 8\alpha=2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha\end{cases}$$

$$\alpha=\frac{2k\pi}{7}\xrightarrow{\text{Max } A}k=3\longrightarrow x=\frac{6\pi}{7}$$

$$\alpha=\frac{2k\pi}{9}\xrightarrow{\text{Max } A}k=4\longrightarrow x=\frac{8\pi}{9}$$

$$\alpha=\frac{(2k+1)\pi}{7}\xrightarrow{\text{Max } A}k=2\longrightarrow x=\frac{5\pi}{7}$$

$$\alpha=\frac{(2k+1)\pi}{9}\xrightarrow{\text{Max } A}k=3\longrightarrow x=\frac{7\pi}{9}$$

$$\longrightarrow\boxed{\text{Max } A=\frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!