امین جلالی

۱۹٫۵ آزمون

(۱) $8\cos x = \tan^2 x + 1 \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow$ ۱ جواب $\quad x = \frac{\pi}{3}, -\frac{\pi}{3}$

(۲) $6(1-\cos^2 x) + 12\cos^2 2x = -2 \Rightarrow -6\cos^2 x + 12\cos^2 2x + 8 = 0 \Rightarrow \begin{cases} \cos^2 x = 1 \\ \cos^2 2x = -1 \end{cases}$

$\Rightarrow \begin{cases} \cos x = \pm 1 \\ 2x = (2k+1)\pi \end{cases} \Rightarrow \begin{cases} x = k\pi \\ x = \frac{(2k+1)\pi}{2} \end{cases} \Rightarrow \cap \Rightarrow \pm\pi$

۲ جواب

(۳) $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta \xrightarrow[2x=\beta]{x=\alpha} \sin(3x) + \sin(-x) + \sin x = 1$

$-\sin(x)$

$\Rightarrow \sin^2 3x = 1 \Rightarrow \begin{cases} 3x = 2k\pi + \frac{\pi}{2} \\ 3x = 2k\pi + \pi - \frac{\pi}{2} \end{cases}$ (جواب نمی‌شود!)

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}$ (جواب) $\rightarrow \frac{15\pi}{6} = \frac{5\pi}{2}$

(۴) $\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} = -\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} \Rightarrow \sin\left(\frac{\pi+4x}{2}\right) + \cos\left(\frac{\pi+8x}{2}\right) = 0$

$\Rightarrow \cos^2 2x - \sin^2 2x = 0 \Rightarrow \cos^2 2x(1 - 2\sin^2 2x) = 0 \Rightarrow \cos \neq 0$

$\sin^2 2x = 2\sin 2x\cos 2x$

$\Rightarrow \sin^2 2x = \frac{1}{2} \Rightarrow 2x = \frac{\pi}{4} \rightarrow x = \frac{\pi}{12}$

$2x = \frac{5\pi}{4} \rightarrow x = \frac{5\pi}{12}$

$\Rightarrow \frac{x+5x}{12} = \frac{\pi}{3} \Rightarrow \tan\frac{2\pi}{3} = -\sqrt{3}$

(۵) $\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \sin 2x = 1 - (\cos x - \sin x)^2 = \frac{1}{3}$

$\Rightarrow m\left(\sqrt{\frac{2}{3}}\right) - 3\sqrt{4}\left(\frac{1}{3}\right) = \sqrt{4} \Rightarrow m = 4$

$m = \frac{2\sqrt{4} \times \sqrt{3}}{\sqrt{2}} =$

(۶)

$\sin(x+\frac{\pi}{4})\cos(\frac{\pi}{4}-x) \Rightarrow \sin^2(x+\frac{\pi}{4})=1 \Rightarrow \sin(x+\frac{\pi}{4})=\pm 1$

$x+\frac{\pi}{4}=2k\pi\pm\frac{\pi}{2} \Rightarrow x=2k\pi\pm\frac{\pi}{3} \Rightarrow$ ۲ جواب دارد

(۲)

(۷)

$\xrightarrow{\div 2} \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2} \Rightarrow \sin\frac{\pi}{4}\sin x+\cos\frac{\pi}{4}\cos x=\cos(\frac{\pi}{6})$

$\Rightarrow \cos(x-\frac{\pi}{4})=\cos\frac{\pi}{6} \Rightarrow x-\frac{\pi}{4}=2k\pi\pm\frac{\pi}{6} \xrightarrow{-\pi\leq x\leq 2\pi} x=\pm\frac{\pi}{6}+\frac{\pi}{4},\ 2\pi-\frac{\pi}{6}+\frac{\pi}{4}$

$\xrightarrow{\text{مجموع}} -\frac{\pi}{6}+\frac{3\pi}{4}+2\pi=\frac{9\pi}{6}$

(۲)

(۸)

$-4\sin x\cos x=1 \Rightarrow 2\sin x\cos x=-\frac{1}{2} \Rightarrow \sin 2x=-\frac{1}{2}$

x باید بین ۰ و ۲π باشد!...

(۱/۵)

$\Rightarrow 2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12} \Rightarrow -\frac{\pi}{12}, \frac{11\pi}{12}$

$n=1$ یا $2 \Rightarrow x=\frac{11\pi}{12}$ و $\frac{23\pi}{12}$

$\Rightarrow 2x=2k\pi-\frac{5\pi}{6} \Rightarrow x=k\pi-\frac{5\pi}{12} \Rightarrow -\frac{5\pi}{12}, \frac{7\pi}{12}$

$n=1$ یا $2 \Rightarrow x=\frac{7\pi}{12}$ و $\frac{19\pi}{12}$

$\xrightarrow{+} \pi$

$S=(\frac{11+19+23+7}{12})\pi=5\pi$

(۹)

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8} \Rightarrow 64\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=1$

$\Rightarrow (8\cos\alpha\cos 2\alpha\cos 4\alpha)^2=1 \xrightarrow{\times\frac{\sin\alpha}{\sin\alpha}} (\frac{\sin 8\alpha}{\sin\alpha})^2=1 \Rightarrow \sin 8\alpha^2=\sin\alpha^2$

$\Rightarrow 8\alpha=k\pi\pm\alpha \Rightarrow \alpha=\frac{k\pi}{7}$ یا $\alpha=\frac{k\pi}{9}$ چون $\sin\neq 0$ $\Rightarrow \max\alpha=\frac{8\pi}{9}$

(۲)

(۱۰)

$\tan 2x=\cot x \Rightarrow \tan 3x=\tan(\frac{\pi}{2}-x) \Rightarrow 3x=k\pi+\frac{\pi}{2}-x$

$\Rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8} \Rightarrow \frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=6\pi$

(۲)

$\sin\left(\frac{3\pi}{2}-x\right)=-\cos x$

$4\sin x(-\cos x)=1 \rightarrow -2(2\sin x\cos x)=1 \rightarrow \sin 2x=-\frac{1}{2}$

$2x=2k\pi-\frac{\pi}{6}$ یا $2k\pi+\frac{7\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12}$ یا $k\pi+\frac{7\pi}{12}$

$0\le x\le 2\pi$

$k=0,1 \rightarrow x=\frac{7\pi}{12}, \frac{19\pi}{12}$

$k=1,2 \rightarrow x=\frac{11\pi}{12}, \frac{23\pi}{12}$

$S=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$