امین جلالی

(1)

$$8\cos x = \tan^2 x + 1 \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \boxed{\cos x = \frac{1}{2}}$$

 → ۲جواب ب $x = \frac{\pi}{3} , -\frac{\pi}{3}$

(2)

$$5(1-\cos^2 x) + 2\cos 3x = -2 \Rightarrow -5\cos^2 x + 2\cos 3x + 7 = 0 \Rightarrow \begin{cases} \cos^2 x = 1 \\ \cos 3x = -1 \end{cases}$$

$$\Rightarrow \begin{cases} \cos x = \pm 1 \\ 3x = (2k+1)\pi \end{cases} \Rightarrow \left.\begin{cases} x = k\pi \\ x = \frac{(2k+1)\pi}{3} \end{cases}\right\} \xrightarrow{\cap} \boxed{\pm\pi}$$

ب ۲جواب دارد که

(3)

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta \xrightarrow[2x=\beta]{x=\alpha} \sin(3x) + \cancel{\sin(-x)} + \sin x = 1$$

$$\cancel{\sin(-x)} = -\sin(x)$$

$$\Rightarrow \sin 3x = 1 \Rightarrow \begin{cases} 3x = 2k\pi + \frac{\pi}{2} \\ 3x = 2k\pi + \cancel{\pi - \frac{\pi}{2}} \end{cases}$$

همونه!

$$\Rightarrow x = \frac{\pi}{6} , \frac{5\pi}{6} , \frac{9\pi}{6} \xrightarrow{\text{مجموع}} \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$$

ب

(4)

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} = -\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} \Rightarrow \sin\left(\frac{\pi+4x}{2}\right) + \cos\left(\frac{\pi+8x}{2}\right) = 0$$

$$\Rightarrow \cos 2x - \cancel{\sin 4x} = 0 \Rightarrow \cos 2x(1 - 2\sin 2x) = 0 \Rightarrow \cos \neq 0$$

$$\cancel{\sin 4x} = 2\sin 2x \cos 2x$$

$$\Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = \frac{\pi}{6} \rightarrow x = \frac{\pi}{12}$$

$$2x = \frac{5\pi}{6} \rightarrow x = \frac{5\pi}{12}$$

$$\xrightarrow{-} \frac{4\pi}{12} = \frac{\pi}{3} \Rightarrow \boxed{\tan\frac{2\pi}{3} = -\sqrt{3}}$$

ب

(5)

$$\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow \sin 2x = 1 - (\cancel{\cos x - \sin x})^2 = \frac{1}{3} \qquad \left(\frac{\sqrt{2}}{\sqrt{3}}\right)$$

$$\Rightarrow m\left(\sqrt{\frac{2}{3}}\right) - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6} \Rightarrow \boxed{m = 6}$$

ب

$$m = \frac{2\sqrt{6} \times \sqrt{3}}{\sqrt{2}} =$$

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$\sin(x+\frac{\pi}{6})\cos(\frac{\pi}{3}-x) \Rightarrow \sin^2(x+\frac{\pi}{6}) = 1 \Rightarrow \sin(x+\frac{\pi}{6}) = \pm 1$

مساوی

$x+\frac{\pi}{6} = 2k\pi \pm \frac{\pi}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3} \Rightarrow$ $\boxed{\text{۲ جواب دارد}}$

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$\xrightarrow{\div 2} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \Rightarrow \sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \cos(\frac{\pi}{4})$

$\Rightarrow \cos(x-\frac{\pi}{6}) = \cos\frac{\pi}{4} \Rightarrow x-\frac{\pi}{6} = 2k\pi \pm \frac{\pi}{4} \xrightarrow{-\pi \leq x \leq 2\pi} x = \pm\frac{\pi}{4}+\frac{\pi}{6},\ 2\pi-\frac{\pi}{4}+\frac{\pi}{6}$

$\xrightarrow{\text{مجموع}} -\frac{\pi}{4}+\frac{3\pi}{6}+2\pi = \boxed{\frac{9\pi}{4}}$

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$-4\sin x\cos x = 1 \Rightarrow 2\sin x\cos x = -\frac{1}{2} \Rightarrow \sin 2x = -\frac{1}{2}$

$\Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12} \Rightarrow -\frac{\pi}{12},\ \frac{11\pi}{12}$

$\Rightarrow 2x = 2k\pi - \frac{5\pi}{6} \Rightarrow x = k\pi - \frac{5\pi}{12} \Rightarrow -\frac{5\pi}{12},\ \frac{7\pi}{12}$

$\xrightarrow{+} \pi$

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$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8} \Rightarrow 64\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = 1$

$\Rightarrow (8\cos\alpha\cos 2\alpha\cos 4\alpha)^2 = 1 \xrightarrow{\times\frac{\sin\alpha}{\sin\alpha}} (\frac{\sin 8\alpha}{\sin\alpha})^2 = 1 \Rightarrow \sin 8\alpha^2 = \sin\alpha^2$

$\Rightarrow 8\alpha = k\pi \pm \alpha \Rightarrow \alpha = \frac{k\pi}{7}$ یا $\alpha = \frac{k\pi}{9}$ چون $\sin \neq 0$ $\Rightarrow \boxed{\max\alpha = \frac{8\pi}{9}}$

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$\tan 3x = \cot x \Rightarrow \tan 3x = \tan(\frac{\pi}{2}-x) \Rightarrow 3x = k\pi + \frac{\pi}{2} - x$

$\Rightarrow x = \frac{k\pi}{4}+\frac{\pi}{8} \Rightarrow \frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8} = \frac{48\pi}{8} = \boxed{6\pi}$