تکلیف ۱۷

۲۰ آفرین

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۱) $8\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1 \rightarrow \cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$ ۲

۲ جواب

۲) $y = 5\sin^2 x + 2\cos(3x) = -2$

$\geq 0 \qquad \geq -2 \longrightarrow y \geq -2 \longrightarrow$ شرط لازم است پس درست $y \geq -2$ نباشد ۰ برابر با $\sin^2 x$ اگر

برای $y=-2$ ، $\sin x = 0$ است

$\sin x = 0 \rightarrow y = 2\cos(3x) = -2 \rightarrow \cos(3x) = -1 \rightarrow 3x = 2k'\pi + \pi$ ۲

$\hookrightarrow x = k\pi$ $\qquad \hookrightarrow x = \left(\frac{2k'+1}{3}\right)\pi$

$\left.\begin{matrix} x = k\pi \\ x = \left(\frac{2k'+1}{3}\right)\pi \end{matrix}\right\} x = -\pi, \pi$ ، ۲ جواب

۳) $2\sin x\cos(2x) + \sin x = 1 \rightarrow \sin x$

$2\sin x(1-2\sin^2 x) + \sin x - 1 = 0 \xrightarrow[t=\sin x]{} 2t - 4t^3 + t - 1 = 0$

$4t^3 - 3t + 1 = 0 = (t+1)(4t^2 - 4t + 1) = (t+1)(2t-1)^2 = 0$ ۲

$t_1 = \sin x_1 = -1 \longrightarrow x_1 = \frac{3\pi}{2}$

$t_2 = \sin x_2 = \frac{1}{2} \longrightarrow x_2 = \frac{\pi}{6}, \frac{5\pi}{6}$ $\Big\} \underset{\oplus}{\Longrightarrow} \frac{5\pi}{2}$

۴) $\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos(+2x)} + \frac{1}{-\sin 4x} = 0$

$\Longrightarrow \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \rightarrow \frac{\sin 4x}{\cos 2x} = \frac{2\sin 2x\cos 2x}{\cos 2x} = 2\sin 2x = 1 \rightarrow \sin 2x = \frac{1}{2}$ ۲

$\hookrightarrow \cos 2x, \sin 4x \neq 0$ $\Big\{ 2x = 2k\pi + \{\frac{\pi}{6}, \frac{5\pi}{6}\} \rightarrow x = k\pi + \{\frac{\pi}{12}, \frac{5\pi}{12}\}$

$|\Delta x| = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = -\sqrt{3}$

۵) $m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$

$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \rightarrow \cos x - \sin x = \sqrt{\frac{2}{3}}$

$\Rightarrow 1 - 2\sin x\cos x = 1 - \sin 2x = \frac{2}{3} \rightarrow \sin 2x = \frac{1}{3}$ ۲

$\Rightarrow \sqrt{\frac{2}{3}}\, m - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \rightarrow m = \frac{2\sqrt{6}}{\frac{\sqrt{2}}{\sqrt{3}}} = 6$

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۸) $\cos(x-\frac{\pi}{12}) \times \underbrace{\sin(x+\frac{\pi}{4})}_{\cos(\frac{\pi}{12}-x)} = \cos^2(x-\frac{\pi}{12}) = 1 \to \sin^2(x-\frac{\pi}{12}) = 0$

$\to x-\frac{\pi}{12} = k\pi \to x = \frac{\pi}{12} + k\pi \longrightarrow x = \frac{\pi}{12}, \frac{13\pi}{12}$ ۲ جواب

۱۰) $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$

$\cos(\frac{\pi}{3})\sin x + \sin(\frac{\pi}{3})\cos(x) = \sin(x+\frac{\pi}{3}) = \sin(\frac{\pi}{4})$

$x+\frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4} + 2k\pi$

$x+\frac{\pi}{3} = +\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4} \to x = -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{23\pi}{12} \to$ جواب $\frac{9\pi}{4}$

۱۱) $4\sin x \sin(\frac{3\pi}{2}-x) = -4\cos x \sin x = 1 \to \sin 2x = -\frac{1}{2}$

$2x = 2k\pi + \frac{7\pi}{6}, \frac{11\pi}{6} \to x = k\pi + \frac{7\pi}{12}, \frac{11\pi}{12} \to x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

۹) $(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)$

$\frac{8\sin^2\alpha}{\sin^2\alpha}\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{\sin^2 8\alpha}{8\sin^2\alpha} = \frac{1}{8} \to \sin^2 8\alpha = \sin^2\alpha$

$\hookrightarrow \sin\alpha \neq 0$

$\begin{cases} 8\alpha = \alpha + 2k\pi \to \alpha = \frac{2k\pi}{7} \\ 8\alpha = \pi - \alpha + 2k'\pi \to \alpha = \frac{k'\pi}{9} \end{cases}$ $\quad k, k' \neq 0, k \neq 7k', 9$

$\begin{cases} \sin^2 x = \sin^2 y \\ \hookrightarrow x-y = k\pi \\ x+y = k'\pi \end{cases}$

$A = \{\frac{\pi}{9}, \frac{2\pi}{7}, \dots, \frac{7\pi}{9}, \frac{8\pi}{7}, \frac{9\pi}{9}\} \to \max\{A\} = \frac{8\pi}{9}$

۱۰) $\tan(3x) = \cot x = \tan(\frac{\pi}{2}-x) \to 3x = \frac{\pi}{2} - x + k\pi$

$\to x = \frac{k\pi}{4} + \frac{\pi}{8} \xrightarrow[[\pi, 2\pi]]{} x = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \to$ ۴ تا