تکلیف ۱۷

رادمهر مقتدری

۱)
$$8\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \to 8\cos^3 x = 1 \to \cos x = \frac{1}{2} \to x = \frac{\pi}{3}, \frac{5\pi}{3}$$

۲ جواب

۲)
$$y = 5\underbrace{\sin^2 x}_{\geq 0} + \underbrace{2\cos(3x)}_{\geq -2} = -2 \to y \geq -2$$

اگر $\sin^2 x$ برابر با ۰ نباشد $y>-2$ هست پس شرط لازم برای $y=-2$ ، $\sin x = 0$ است

$$\sin x = 0 \to y = 2\cos(3x) = -2 \to \cos(3x) = -1 \to 3x = 2k'\pi + \pi \to x = \left(\frac{2k'+1}{3}\right)\pi$$

$$\to x = k\pi$$

$$\left.\begin{matrix} x = k\pi \\ x = \left(\frac{2k'+1}{3}\right)\pi \end{matrix}\right\} x = -\pi, \pi$$ ، ۲ جواب

۳)
$$2\sin x\cos(2x) + \sin x = 1 \to \sin x$$

$$2\sin x(1-2\sin^2 x) + \sin x - 1 = 0 \xrightarrow[t=\sin x]{} 2t - 4t^3 + t - 1 = 0$$

$$4t^3 - 3t + 1 = 0 = (t+1)(4t^2 - 4t + 1) = (t+1)(2t-1)^2 = 0$$

$$\left.\begin{matrix} t_1 = \sin x_1 = -1 \longrightarrow x_1 = \frac{3\pi}{2} \\ t_2 = \sin x_2 = \frac{1}{2} \longrightarrow x_2 = \frac{\pi}{6}, \frac{5\pi}{6} \end{matrix}\right\} \xRightarrow{\oplus} \frac{5\pi}{2}$$

۴)
$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos(+2x)} + \frac{1}{-\sin 4x} = 0$$

$$\Rightarrow \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \to \frac{\sin 4x}{\cos 2x} = \frac{2\sin 2x\cos 2x}{\cos 2x} = 2\sin 2x = 1 \to \sin 2x = \frac{1}{2}$$

$$\to \cos 2x, \sin 4x \neq 0 \quad \Big\} \quad 2x = 2k\pi + \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} \to x = k\pi + \left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$$

$$|\Delta x| = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3} \to \tan(2x) = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

۵)
$$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$$

$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \to \cos x - \sin x = \sqrt{\frac{2}{3}}$$

$$\Rightarrow 1 - 2\sin x\cos x = 1 - \sin 2x = \frac{2}{3} \to \sin 2x = \frac{1}{3}$$

$$\Rightarrow \sqrt{\frac{2}{3}}\, m - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \to m = \frac{2\sqrt{6}}{\frac{\sqrt{2}}{\sqrt{3}}} = 6$$

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۶)

$$\cos(x-\frac{\pi}{3}) \times \underbrace{\sin(x+\frac{\pi}{6})}_{\cos(\frac{\pi}{3}-x)} = \cos^2(x-\frac{\pi}{3}) = 1 \to \sin^2(x-\frac{\pi}{3}) = 0$$

$$\to x-\frac{\pi}{3} = k\pi \to x = \frac{\pi}{3}+k\pi \to x = \frac{\pi}{3}, \frac{4\pi}{3}$$ ۲ جواب

۷)

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos(\frac{\pi}{3})\sin x + \sin(\frac{\pi}{3})\cos(x) = \sin(x+\frac{\pi}{3}) = \sin(\frac{\pi}{4})$$

$$x+\frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4} + 2k\pi$$

$$x+\frac{\pi}{3} = +\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4} \to x = -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{23\pi}{12} \to \frac{9\pi}{4}$$ جواب

۸)

$$4\sin x \sin(\frac{3\pi}{2}-x) = -4\cos x \sin x = 1 \to \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi + \frac{7\pi}{6}, \frac{11\pi}{6} \to x = k\pi + \frac{7\pi}{12}, \frac{11\pi}{12} \to x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$\to 5\pi$ جمع

۹)

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)$$

$$\frac{8\sin^2\alpha}{\sin^2\alpha}\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{\sin^2 8\alpha}{8\sin^2\alpha} = \frac{1}{8} \to \sin^2 8\alpha = \sin^2\alpha$$

$\sin\alpha \neq 0 \to 8\alpha = \alpha + k\pi \to \alpha = \frac{k\pi}{7}$

$k, k' \neq 0, k \neq 7, k' \neq 9 \quad 8\alpha = -\alpha + k'\pi \to \alpha = \frac{k'\pi}{9}$

$\sin^2 x = \sin^2 y \to x-y = k\pi, \; x+y = k'\pi$

$$A = \{\frac{\pi}{9}, \frac{\pi}{7}, \dots, \frac{7\pi}{9}, \frac{6\pi}{7}, \frac{8\pi}{9}\} \to \max\{A\} = \frac{8\pi}{9}$$

۱۰)

$$\tan(3x) = \cot x = \tan(\frac{\pi}{2}-x) \to 3x = \frac{\pi}{2} - x + k\pi$$

$$\to x = \frac{k\pi}{4} + \frac{\pi}{8} \xrightarrow[{[\pi, 2\pi]}]{} x = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \to 6\pi$$