تکلیف ۱۷ ۱۷ مرادساعی

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$8\cos - \frac{\sin^2}{\cos^2} = 1 \Rightarrow \frac{8\cos^3 - (1-\cos^2)}{\cos^2} = 1 \Rightarrow 8\cos^3 + \cos^2 - 1 = \cos^2$

$\Rightarrow 8\cos^3 = 1 \Rightarrow \cos^3 = \frac{1}{8} \Rightarrow \cos = \frac{1}{2} \Rightarrow$ $\boxed{\text{جواب ۲}}$

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$5\sin^2(x) + 2\cos(2x) = -2 \Rightarrow \sin x = 0$, $\Rightarrow \cos x = -1$ $\Big\} \Rightarrow x = \pi, -\pi \Rightarrow$ جواب ۲

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$\sin(2(\cos^2\alpha + 1)) = 1 \Rightarrow 2\sin\left(\frac{\cos^2\alpha + 1}{2}\right) \xrightarrow{\cos^2\alpha + 1 = 1 - 2\sin^2} $

$\sin(3 - 4\sin^2) = 1 \Rightarrow 3\sin - 4\sin^3 = 1 \Rightarrow \sin 3\alpha = 1 \Rightarrow \alpha = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}$

$\Rightarrow$ جواب ۳ $\Rightarrow$ جمع $= \frac{15\pi}{6}$

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$\sin\left(\frac{\pi}{2} + 2x\right) = \cos 2x \qquad \cos\left(\frac{\pi}{2} + 2x\right) = -\sin 2x$

$\Rightarrow \frac{-2\sin 2x\cos 2x + \cos 2x}{-2\sin 2x\cos^2 2x} = \frac{\cancel{\cos 2x}(-2\sin 2x + 1)}{-2\cancel{\cos^2 2x}\sin 2x} = 0$

$\Rightarrow -2\sin 2x + 1 = 0 \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12} \Rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\Rightarrow \tan 2\frac{\pi}{3} = -\sqrt{3}$

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$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{1}{\sqrt{2}} \Rightarrow \cos - \sin = \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \frac{m\sqrt{2}}{\sqrt{3}} - \sqrt{6} = \sqrt{6}$

$\Rightarrow \frac{\sqrt{2}\,m}{\sqrt{3}} = 2\sqrt{6} \Rightarrow m = 6$

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$\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) = 1$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)^2 = 1 \Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \pm 1$

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$\Rightarrow \sqrt{1}\sin\left(x + \frac{\pi}{6}\right) = \pm 1 \Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3} \Rightarrow \frac{\pi}{3} + \frac{4\pi}{3} =$

۲ جواب

(7

$\sin x + \sqrt{3}\cos x = \sqrt{2} \Rightarrow \sqrt{4}\sin(x+\frac{\pi}{3}) = \sqrt{2}$

$\Rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \Rightarrow$ $x = \frac{3\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12}$, $x = \frac{\pi}{4} - \frac{\pi}{3} = \frac{-\pi}{12}$, $x = \frac{23\pi}{12}$ $\Big\}$ $\frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12}$ [illegible]

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(8

$\sin(\frac{3\pi}{2} - x) = -\cos x \Rightarrow -4\sin\cos x = -2\sin 2x = 1 \Rightarrow \sin 2x = -\frac{1}{2}$

1,5

$\Rightarrow 2x = \frac{7\pi}{6}, \frac{11\pi}{6} \to x = \frac{7\pi}{12}, \frac{11\pi}{12} \to \frac{11\pi}{12} - \frac{7\pi}{12} = \frac{\pi}{3}$

(9

15

رسید؟ $\frac{8\pi}{9}$

(10

1

$\tan(2\alpha)\tan\alpha = 1 \Rightarrow \frac{\sin 2\alpha}{\cos 2\alpha} \cdot \frac{\sin\alpha}{\cos\alpha} = 1 \Rightarrow \sin(2\sin - 4\sin^3) = \cos(4\cos^3 - 3\cos)$

$\Rightarrow \sin = -\cos \Rightarrow \frac{7\pi}{4}$

$\tan 3x = \frac{1}{\tan x} = \cot x \to \tan 3x = \tan(\frac{\pi}{2} - x)$ 10

$3x = k\pi + \frac{\pi}{2} - x \to 4x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{4} + \frac{\pi}{8}$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$S = (\frac{9+11+13+15}{8})\pi = \frac{48\pi}{8} = \boxed{6\pi}$

9

$1+\cos 2x = 2\cos^2 x$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$ $\rightarrow \boxed{\max A = \frac{8\pi}{9}}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!

8

$\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$

$4\sin x(-\cos x) = 1 \rightarrow -2(2\sin x\cos x) = 1 \rightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6}$ یا $2k\pi + \frac{7\pi}{6} \rightarrow x = \overset{2}{k\pi - \frac{\pi}{12}}$ یا $\overset{1}{k\pi + \frac{7\pi}{12}}$

$$\xrightarrow{0\leq x\leq 2\pi} \begin{cases} \overset{1}{k=0,1} \rightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ \underset{2}{k=1,2} \rightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$