۱۶ لطفا مرتب تر بنویس!

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$\cos(x - \frac{\pi}{2}) = \cos(\frac{\pi}{2} - x) = \sin(\frac{\pi}{2} - \frac{\pi}{2} + x) = \sin(\frac{\pi}{4} + x)$

$x = \frac{\pi}{4}$

$\sin^2(x + \frac{\pi}{4}) = 1$

$\sin(x + \frac{\pi}{4}) = 1 \Rightarrow x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{2} \Rightarrow x = 2k\pi + \frac{\pi}{4}$ ۶

$\sin(x + \frac{\pi}{4}) = -1 \quad x + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{2} \Rightarrow x = 2k\pi + \frac{5\pi}{4}$

$\frac{\pi}{4} + \frac{3\pi}{4} = \pi$

۱,۷۵

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{3}}{2}$

$\sin(x + \frac{\pi}{3}) = \sin(\frac{\pi}{3})$

$x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \Rightarrow x = 2k\pi - \frac{\pi}{12}$

$x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \Rightarrow x = 2k\pi + \frac{5\pi}{12}$

$-\frac{\pi}{12} + \frac{5\pi}{12} + \frac{\pi}{4} = \frac{15\pi}{12}$, $\frac{5\pi}{6}$

$2\pi - \frac{\pi}{12} = \frac{23\pi}{12}$

$(\frac{23}{12} + \frac{5}{12} - \frac{1}{12})\pi = \frac{27\pi}{12}$ ۷

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$\sin(\frac{3\pi}{2} - x) = -\cos x$

$\sin 2x = -\frac{1}{2} \Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \frac{7\pi}{6} \Rightarrow x = k\pi + \frac{7\pi}{12}$ ۸

$\frac{23\pi}{12} + \frac{11\pi}{12} + \frac{7\pi}{12} + \frac{19\pi}{12} = \frac{54\pi}{12} = \frac{27\pi}{6}$, $\frac{9\pi}{2}$

$\frac{60\pi}{12} = 5\pi$

۱,۷۵

$\cos 2x = \frac{1}{2} \quad x = 2k\pi + \frac{\pi}{3}$

$\cos x = \frac{1}{2} \quad x = k\pi + \frac{\pi}{6}$

$\cos x = 1$

$x = \frac{2k\pi}{3} + \frac{\pi}{12}$

$\frac{7\pi}{12}$ ۹

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$\tan 3x = \frac{1}{\tan x} \Rightarrow \tan 3x = \cot x \Rightarrow \tan 3x = \tan(\frac{\pi}{2} - x)$

$3x = k\pi + \frac{\pi}{2} - x \Rightarrow 4x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$ ۱۰

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(۱)

$$\Lambda\cos x = 1+\tan^2 x \rightarrow \Lambda\cos x = \frac{1}{\cos^2 x} \rightarrow \Lambda\cos^3 x = 1$$

$$\cos^3 x = \frac{1}{\Lambda} \rightarrow \cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{0 \le x < 2\pi} \boxed{x = \frac{\pi}{3} \text{ یا } \frac{5\pi}{3}}$$

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$$\cos 2x = 1 - 2\sin^2 x \quad \text{(فرمول طلایی)}$$

$$2\sin x(1-2\sin^2 x) + \sin x = 1 \rightarrow 3\sin x - 4\sin^3 x = 1$$

$$\sin 3x = 1 \rightarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x < 2\pi}$$

$$k=0 \rightarrow x = \frac{\pi}{6} \;,\; k=1 \rightarrow x = \frac{5\pi}{6} \;,\; k=2 \rightarrow x = \frac{9\pi}{6}$$

$$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$$

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$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{2} \rightarrow \cos x - \sin x = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos x - \sin x = \frac{\sqrt{3}}{\sqrt{2}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{2} \rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{2}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \rightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 2\sqrt{6}(\sin 2x) = \sqrt{6} \rightarrow \frac{\sqrt{6}}{2}m - 2\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{2}m - \sqrt{6} = \sqrt{6} \rightarrow \frac{m\sqrt{6}}{2} = 2\sqrt{6} \rightarrow \boxed{m = 4}$$

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$$\cos(x - \frac{\pi}{2}) = \cos(\frac{\pi}{2} - x)$$

$$\frac{\pi}{2} - x + x + \frac{\pi}{4} = \frac{3\pi}{4} \rightarrow \cos(\frac{\pi}{2} - x) = \sin(x + \frac{\pi}{4})$$

$$\sin^2(x+\frac{\pi}{4}) = 1 \rightarrow \sin(x+\frac{\pi}{4}) = \pm 1 \rightarrow x + \frac{\pi}{4} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{4} \xrightarrow{0 \le x < 2\pi} k=0 \rightarrow x = \frac{\pi}{4} \;,\; k=1 \;,\; x = \frac{5\pi}{4}$$

معادله ۲ جواب دربازه دارد

(۸) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ \sin x + \sin 45^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi \\ x + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{\pi}{2} \end{cases} \xrightarrow{0 \le x \le 2\pi}$$

$k=0 \to x=0$

$k=1 \to x=2\pi$

$k=0 \to x=\frac{\pi}{2}$

$k=1 \to x=\frac{3\pi}{2}$

$$S = 0 + 2\pi + \frac{\pi}{2} + \frac{3\pi}{2} = 4\pi$$

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(۹)

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \longrightarrow -2(2\sin x\cos x) = 1 \longrightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \quad \text{یا} \quad 2k\pi + \frac{7\pi}{6} \longrightarrow x = k\pi - \frac{\pi}{12} \quad \text{یا} \quad k\pi + \frac{7\pi}{12}$$

$$0 \le x \le 2\pi \begin{cases} k=1 \longrightarrow x = \frac{11\pi}{12}, \frac{19\pi}{12} \\ k=2 \longrightarrow x = \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi$$

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$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	۴	۵	۶	۷
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = (9+11+13+15)\frac{\pi}{8} = \frac{48\pi}{8} = 6\pi$$

$1+\cos 2x = 2\cos^2 x$

$\begin{cases} 1+\cos 2\alpha = 2\cos^2 \alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}$? $\rightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$

$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{}} \cos\alpha \cos 2\alpha \cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha}$ ✱

$(\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha})\cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}(\underbrace{\sin 2\alpha \cos 2\alpha}_{\frac{1}{2}\sin 4\alpha})\cos 4\alpha = \pm\frac{\sin\alpha}{8}$

$\frac{1}{2}\times\frac{1}{2}(\underbrace{\sin 4\alpha \cos 4\alpha}_{\frac{1}{2}\sin 8\alpha}) = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$

$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \quad \text{یا} \quad 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \quad \text{یا} \quad 2k\pi + \pi + \alpha \end{cases}$

$\alpha = \frac{2k\pi}{7} \xrightarrow{MAX\ A} k=3 \rightarrow x = \frac{6\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{MAX\ A} k=4 \rightarrow x = \frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAX\ A} k=3 \rightarrow x = \frac{6\pi}{7}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAX\ A} k=4 \rightarrow x = \frac{9\pi}{9}$

$\rightarrow \boxed{MAX\ A = \frac{8\pi}{9}}$

✱ A نمی‌تواند برابر خود π باشد چون طبق ✱ $\sin\alpha$ را صفر در نظر گرفته‌ایم!