۱-

$8\cos x - \tan^2 x = 1 \rightarrow 8\cos x = \dfrac{1+\tan^2 x}{\;} = \dfrac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1 \rightarrow \cos^3 x = \dfrac{1}{8} \rightarrow \cos x = \dfrac{1}{2}$

$\rightarrow x = 2k\pi \pm \dfrac{\pi}{3} \rightarrow$ (دو جواب در بازه‌ی $[0, 2\pi]$)

۲-

$5\sin^2(x) + 2\cos(2x) = -2$

۳-

$2\sin x\cos 2x + \sin x = 1$

$\rightarrow 2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow -4\sin^3 x + 3\sin x - 1 = 0$

$\rightarrow (\sin x + 1)(-4\sin^2 x + 4\sin x - 1) = 0 \begin{cases} \sin x = -1 \rightarrow x = 2k\pi + \dfrac{3\pi}{2} \\ \sin x = \dfrac{1}{2} \Rightarrow x = 2k\pi + \dfrac{\pi}{6},\ 2k\pi + \dfrac{5\pi}{6} \end{cases} \Rightarrow \dfrac{\pi}{6} + \dfrac{5\pi}{6} + \dfrac{3\pi}{2} = \dfrac{5\pi}{2}$

۴-

$\dfrac{1}{\underbrace{\sin\left(\frac{\pi + 4x}{2}\right)}_{\cos 2x}} + \dfrac{1}{\underbrace{\cos\left(\frac{\pi + 4x}{2}\right)}_{-\sin 2x}} = 0 \rightarrow \dfrac{\cos 2x - \sin 2x}{-\cos 2x \sin 2x} = 0 \rightarrow \cos 2x - \overset{2\sin 2x\cos 2x}{\cancel{\sin 4x}} = 0 \rightarrow \cos 2x(1 - 2\sin 2x) = 0$

$2\alpha = \dfrac{2\pi}{3} \rightarrow \tan 2\alpha = -\sqrt{3}$

$\rightarrow \begin{cases} \cos 2x = 0 \quad \times \quad \text{مخرج صفر نباید باشد} \\ \sin 2x = \dfrac{1}{2} \rightarrow 2x = 2k\pi + \dfrac{\pi}{6},\ 2k\pi + \dfrac{5\pi}{6} \rightarrow x = k\pi + \dfrac{\pi}{12},\ k\pi + \dfrac{5\pi}{12} \end{cases} \rightarrow \dfrac{\pi}{12}, \dfrac{5\pi}{12} \rightarrow \alpha = \dfrac{4\pi}{12} = \dfrac{\pi}{3}$

۵-

$\cos\left(x + \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}} \rightarrow \cos\left(x + \dfrac{\pi}{4}\right) = \sin\left(-x + \dfrac{\pi}{4}\right) = -\sin\left(x - \dfrac{\pi}{4}\right) \rightarrow -\sin\left(x - \dfrac{\pi}{4}\right) = \dfrac{-(\sin x - \cos x)}{\sqrt{2}}$

$\rightarrow \dfrac{\cos x - \sin x}{\sqrt{2}} = \dfrac{1}{\sqrt{3}} \rightarrow \boxed{\cos x - \sin x = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{6}}{3}} \rightarrow \dfrac{6}{9} = \underbrace{\cos^2 + \sin^2}_{1} - \underbrace{2\sin x\cos x}_{-\sin 2x}$

$\rightarrow \boxed{\sin 2x = \dfrac{1}{3}}$

$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6} \rightarrow \dfrac{m\sqrt{6}}{3} - \cancel{\dfrac{3\sqrt{6}}{3}} = \sqrt{6} \rightarrow \dfrac{m\sqrt{6}}{3} = 2\sqrt{6} \rightarrow m = 6$

۶-

$\sin\left(x + \dfrac{\pi}{6}\right)\underbrace{\cos\left(x - \dfrac{\pi}{3}\right)}_{\cos\alpha = \cos -\alpha} = 1 \rightarrow \sin\left(x + \dfrac{\pi}{6}\right)\underbrace{\cos\left(\dfrac{\pi}{3} - x\right)}_{\sin\left(x + \frac{\pi}{6}\right)} = 1 \rightarrow \sin^2\left(x + \dfrac{\pi}{6}\right) = 1 \rightarrow \sin\left(x + \dfrac{\pi}{6}\right) = \pm 1$

$\rightarrow x + \dfrac{\pi}{6} = k\pi + \dfrac{\pi}{2} \rightarrow x = k\pi + \dfrac{\pi}{3}$

$\rightarrow$ ۲ جواب در بازه‌ی $[0, 2\pi]$

[illegible]

$\sin x + \sqrt{3}\cos x = \sqrt{2} \to \sqrt{\cancel{1+(\sqrt{3})^2}}\,\sin(x+\alpha) \to 2\sin(x+\frac{\pi}{3}) = \sqrt{2} \to \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$ ۷-

$\tan\alpha = \sqrt{3} \to \alpha = \frac{\pi}{3}$

$\to x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4},\ 2k\pi + \frac{3\pi}{4}$

$\to x = 2k\pi - \frac{\pi}{12},\ 2k\pi + \frac{5\pi}{12}$ — ۳ جواب در بازه‌ی $[-\pi, 2\pi]$

$4\sin x \sin(\frac{3\pi}{2} - x) = 1 \to -4\sin x\cos x = 1 \to -2\sin 2x = 1 \to \sin 2x = -\frac{1}{2}$ ۸-

$\to 2x = 2k\pi + \frac{7\pi}{6},\ 2k\pi + \frac{11\pi}{6} \to x = k\pi + \frac{7\pi}{12},\ k\pi + \frac{11\pi}{12}$ — ۴ جواب در بازه‌ی $[0, 2\pi]$

$(1+\cos(2\alpha))(1+\cos(4\alpha))(1+\cos(8\alpha)) = \frac{1}{8} \xrightarrow{1+\cos 2x = 2\cos^2 x} (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$ ۹-

$\to 8\left[(\cos\alpha)(\cos 2\alpha)(\cos 4\alpha)\right]^2 = \frac{1}{8} \to (\cos\alpha)(\cos 2\alpha)(\cos 4\alpha) = \pm\frac{1}{8} \xrightarrow{\times \sin\alpha} \frac{\sin 8\alpha}{8} = \pm\frac{1}{8}\sin\alpha$

$\begin{cases} \sin 8\alpha = \sin\alpha \to \alpha = \begin{cases} \alpha = \frac{2}{7}k\pi \\ \alpha = \frac{2}{9}k\pi + \frac{\pi}{9} \end{cases} \\ \sin 8\alpha = -\sin\alpha = \sin(-\alpha) \to \alpha = \begin{cases} \alpha = \frac{2k\pi}{9} \\ \alpha = \frac{2}{7}k\pi + \frac{\pi}{7} \end{cases} \end{cases}$ — $Max = \pi$

$[0, \pi]$ بازه‌ی

$\tan(3x)\tan(x) = 1 \to \tan 3x = \cot x \to 3x = k\pi + \frac{\pi}{2} - x \to 4x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{4} + \frac{\pi}{8}$ ۱۰

$\to \frac{5\pi}{8}, \frac{7\pi}{8}$ — مجموع $= \frac{3}{2}\pi$