صالح صالحی — به نام آنکه جان را فکرت آموخت — دوازدهم - البرز B - تکلیف ۱۷

۱)

$8\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \to 8\cos^3 x = 1 \to \cos x = \frac{1}{2}$

$x = 2k\pi \pm \frac{\pi}{3} \to \frac{\pi}{3}, \frac{5\pi}{3}$ ۲ جواب ✓ ۲

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۲)

$2\sin^2\alpha = 1-\cos2\alpha \to 5\sin^2\alpha = 5-5\cos2\alpha \Rightarrow 5-5\cos2\alpha+2\cos(3\alpha)$

$5\cos2\alpha - 2\cos3\alpha = 7$ ادامه؟

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۳)

$2\sin x(2\cos^2 x-1)+\sin x = \sin x(4\cos^2 x-1) = \sin x(2\cos x-1)(2\cos x+1)$

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۴)

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos2x} - \frac{1}{\sin4x} = \frac{\sin4x-\cos2x}{\cos2x\sin4x}$

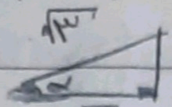
$\frac{2\sin2x\cos2x-\cos2x}{2\cos^2 2x\sin2x} = \frac{\cos2x(2\sin2x-1)}{2\cos^2 2x\sin2x} = \frac{2\sin2x-1}{\sin4x} = 0 \xrightarrow{\sin4x\neq0}$

۱/۵

$\sin2x = \frac{1}{2}$ $\begin{cases} 2x = 2k\pi+\frac{\pi}{6} \to x = k\pi+\frac{\pi}{6} \\ 2x = 2k\pi+\frac{5\pi}{6} \to x = k\pi+\frac{5\pi}{6} \end{cases}$ $4x \neq k\pi \to x \neq \frac{k\pi}{4}$

$[0,\pi]$ $\begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases}$ $\ominus$ $\frac{2\pi}{3} \to \tan\frac{4\pi}{3} = \sqrt{3}$

سطح ۲ بالا

۵) $\sin\left(\frac{\pi}{2}-\left(x+\frac{\pi}{4}\right)\right) = \sin(\underbrace{\frac{\pi}{4}-x}_{\alpha}) = \frac{1}{\sqrt3}$  $\to \sin 2\alpha = 2\cos\alpha\sin\alpha$

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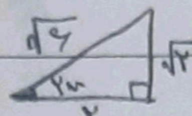
$= \frac{2}{\sqrt6} = \sin\left(\frac{\pi}{2}-2x\right) = \cos 2x = \frac{2}{\sqrt6} \to$  $\Rightarrow \sin 2x = \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}}$

$m(\cos x - \sin x) = \sqrt6 + \left(3\sqrt6 \times \frac{1}{\sqrt3}\right) = \sqrt6 + 3\sqrt2 =$

۶) $\cos\left(x-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}-x\right) = \sin\left(\frac{\pi}{6}+x\right) \Rightarrow \sin^2\left(\frac{\pi}{6}+x\right) = 1 \to \sin\left(\frac{\pi}{6}+x\right) = \pm1$

$\Rightarrow x+\frac{\pi}{6} = k\pi+\frac{\pi}{2} \to x = k\pi+\frac{\pi}{3} \Rightarrow \frac{\pi}{3}, \frac{4\pi}{3}$ دو تا جواب

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۷) $\sin x + \sqrt3\cos x = 2\sin\left(x+\frac{\pi}{3}\right) = \sqrt2, \quad \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt2}{2} \qquad \tan\alpha = \sqrt3, \ \alpha = \frac{\pi}{3}$

$x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \to x = 2k\pi-\frac{\pi}{12} \to -\frac{\pi}{12}, \frac{23\pi}{12}$

$x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4} \to x = 2k\pi+\frac{5\pi}{12} \to \frac{5\pi}{12}$

مجموع $\Leftarrow \ = \frac{9\pi}{4}$

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۸) $-2\sin 2x = 1, \quad \sin 2x = -\frac{1}{2}$

$2x = 2k\pi+\frac{7\pi}{6} \Rightarrow x = k\pi+\frac{7\pi}{12}$

$2x = 2k\pi+\frac{11\pi}{6} \Rightarrow x = k\pi-\frac{\pi}{12}$

$\to \frac{7\pi}{12}, \frac{19\pi}{12}, \frac{11\pi}{12}, \frac{23\pi}{12} \to \Sigma = 5\pi$

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۹) $(\sin^2\alpha+\cos^2\alpha+\cos^2\alpha-\sin^2\alpha)(\sin^2 2\alpha+\cos^2 2\alpha+\cos^2 2\alpha-\sin^2 2\alpha)(\sin^2 4\alpha+\cos^2 4\alpha+\cos^2 4\alpha-\sin^2 4\alpha)$

$= (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$

$\to (\cos 2\alpha\cos\alpha\cos 4\alpha)^2 = \frac{1}{64} \to \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{1}{8} \xrightarrow{\times\sin\alpha}$ ۱/۵

$\frac{1}{8}\sin 8\alpha = \frac{\sin\alpha}{8} \to \sin 8\alpha = \sin\alpha, \quad 8\alpha = 2k\pi+\alpha \to \alpha = \frac{2k\pi}{7}$

$8\alpha = 2k\pi+\pi-\alpha \to \alpha = \frac{2k\pi}{9}+\frac{\pi}{9}$

$\to \max = \frac{\pi}{2}$

- ۲

$\cos 3x = 2\cos^2 \frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2\left(2\cos^2 \frac{3x}{2} - 1\right) = -2 \rightarrow 5\sin^2 x + 4\cos^2 \frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \leq x \leq \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2 \frac{3x}{2} = 0 \rightarrow \cos \frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{-\pi \leq x \leq \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک دو مجموعه جواب $\{\pi, -\pi\}$ است

۱۰

$\tan x = \cot 3x = \tan\left(\frac{\pi}{2} - 3x\right) = \tan x \rightarrow x = k\pi + \frac{\pi}{2} - 3x$

$\rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8} \rightarrow [\pi, 2\pi] \rightarrow \pi + \frac{\pi}{8}, \frac{5\pi}{8} + \frac{\pi}{8}, \frac{3\pi}{2} + \frac{\pi}{8}$

$\frac{7\pi}{4} + \frac{\pi}{8}$

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$\Rightarrow \sum = 6\pi$

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$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

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$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \rightarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \leq x \leq 2\pi}$

$k = 0 \rightarrow x = \frac{\pi}{6}$, $k = 1 \rightarrow x = \frac{5\pi}{6}$, $k = 2 \rightarrow x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

(۴)

$$\begin{cases} \sin(2x+\frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2}+4x) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$$\cos 2x - 2\sin 2x \cos 2x = 0 \rightarrow \cos 2x(1-2\sin 2x) = 0 \rightarrow \begin{cases} \cos 2x = 0 & \times \\ \sin 2x = \frac{1}{2} & \checkmark \end{cases}$$

★ اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12} \text{ یا } k\pi + \frac{5\pi}{12}$$

$$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$$

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(۵)

$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \rightarrow \cos x - \sin x = \frac{\sqrt{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \rightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \rightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \rightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \rightarrow \boxed{m = 6}$$

(9)

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} \bigstar$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!