صالح صالحی — به نام آنکه جان را فکرت آموخت — دوازدهم، الیبرا B، تکلیف ۱۷

(۱)

$8\cos x = 1+\tan^2 x = \dfrac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1 \rightarrow \cos x = \dfrac{1}{2}$

$x = 2k\pi \pm \dfrac{\pi}{3} \rightarrow \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ ۲ جواب

(۲)

$2\sin^2\alpha = 1-\cos 2\alpha \rightarrow 5\sin^2\alpha = 5 - 5\cos 2\alpha \Rightarrow 5 - 5\cos 2\alpha + 2\cos(3\alpha)$

$5\cos 2\alpha - 2\cos 3\alpha = 7$

(۳)

$2\sin x(2\cos^2 x - 1) + \sin x = \sin x(4\cos^2 x - 1) = \sin x(2\cos x - 1)(2\cos x + 1)$

(۴)

$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x}$$

$$\frac{2\sin 2x\cos 2x - \cos 2x}{2\cos^2 2x \sin 2x} = \frac{\cos 2x(2\sin 2x - 1)}{2\cos^2 2x \sin 2x} = \frac{2\sin 2x - 1}{\sin 4x} = 0 \xrightarrow{\sin 4x \neq 0}$$

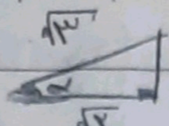
$$\sin 2x = \frac{1}{2} \begin{cases} 2x = 2k\pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{6} \\ 2x = 2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{5\pi}{6} \end{cases} \quad 4x \neq k\pi \rightarrow x \neq \frac{k\pi}{4}$$

$$[0,\pi] \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases} \ominus \ \frac{2\pi}{3} \rightarrow \tan\frac{4\pi}{3} = \sqrt{3}$$

پاسخ سؤالات

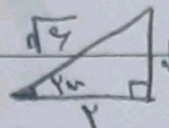
(۵)

$\sin\left(\frac{\pi}{2}-\left(x+\frac{\pi}{4}\right)\right) = \sin(\underbrace{\frac{\pi}{4}-x}_{\alpha}) = \frac{1}{\sqrt3}$



$\to \sin2\alpha = 2\cos\alpha\sin\alpha$

$= \frac{2}{\sqrt6} = \sin\left(\frac{\pi}{2}-2x\right) = \cos2x = \frac{2}{\sqrt6} \to$



$\Rightarrow \sin2x = \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}}$

$m(\cos x-\sin x) = \sqrt6 + \left(3\sqrt6 \times \frac{1}{\sqrt3}\right) = \sqrt6 + 3\sqrt2 =$

(۶)

$\cos\left(x-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}-x\right) = \sin\left(\frac{\pi}{6}+x\right) \Rightarrow \sin^2\left(\frac{\pi}{6}+x\right) = 1 \to \sin\left(\frac{\pi}{6}+x\right) = \pm1$

$\Rightarrow x+\frac{\pi}{6} = k\pi+\frac{\pi}{2} \to x = k\pi+\frac{\pi}{3} \Rightarrow \frac{\pi}{3}, \frac{4\pi}{3}$ ، دو تا جواب

(۷)

$\sin x + \sqrt3\cos x = 2\sin\left(x+\frac{\pi}{3}\right) = \sqrt2 \to \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt2}{2}$ $\quad \tan\alpha = \sqrt3 \to \alpha = \frac{\pi}{3}$

$x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \to x = 2k\pi-\frac{\pi}{12} \to -\frac{\pi}{12}, \frac{23\pi}{12}$

$x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4} \to x = 2k\pi+\frac{5\pi}{12} \to \frac{5\pi}{12}$

مجموع $\Rightarrow \; = \frac{9\pi}{4}$

(۸)

$-2\sin2x = 1 \to \sin2x = -\frac{1}{2}$

$\begin{cases} 2x = 2k\pi+\frac{7\pi}{6} \Rightarrow x = k\pi+\frac{7\pi}{12} \\ 2x = 2k\pi+\frac{11\pi}{6} \Rightarrow x = k\pi-\frac{\pi}{12} \end{cases}$

$\to \frac{7\pi}{12}, \frac{19\pi}{12}, \frac{11\pi}{12}, \frac{23\pi}{12} \to \sum = 5\pi$

(۹)

$(\sin^2\alpha+\cos^2\alpha+\cos^2\alpha-\sin^2\alpha)(\sin^2 2\alpha+\cos^2 2\alpha+\cos^2 2\alpha-\sin^2 2\alpha)(\sin^2 4\alpha+\cos^2 4\alpha+\cos^2 4\alpha-\sin^2 4\alpha)$

$= (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$

$\to (\cos2\alpha\cos\alpha\cos4\alpha)^2 = \frac{1}{64} \to \cos\alpha\cos2\alpha\cos4\alpha = \frac{1}{8} \xrightarrow{\times\sin\alpha}$

$\frac{1}{8}\sin8\alpha = \frac{\sin\alpha}{8} \to \sin8\alpha = \sin\alpha \to 8\alpha = 2k\pi+\alpha \to \alpha = \frac{2k\pi}{7}$

$8\alpha = 2k\pi+\pi-\alpha \to \alpha = \frac{2k\pi}{9}+\frac{\pi}{9}$

$\to \max = \frac{\pi}{2}$

$\tan x = \cot ۳x = \tan\left(\frac{\pi}{۲} - ۳x\right) = \tan x \Rightarrow x = k\pi + \frac{\pi}{۲} - ۳x$ ۱۰

$\rightarrow ۴x = k\pi + \frac{\pi}{۲} \rightarrow x = \frac{k\pi}{۴} + \frac{\pi}{۸} \rightarrow [\pi, ۲\pi] \rightarrow \pi + \frac{\pi}{۸}, \frac{۵\pi}{۸} + \frac{\pi}{۸}, \frac{۳\pi}{۲} + \frac{\pi}{۸}$

$\frac{۷\pi}{۴} + \frac{\pi}{۸}$

$\Rightarrow \sum = ۶\pi$