۵ تا تمرین رفع اشکال بنویسید !

نام و نام خانوادگی ......علیرضا احسان‌پور...... پاسخنامه تشریحی تکلیف شماره .... کلاس	
$8\cos = 1 + \tan^2 \to 8\cos = \frac{1}{\cos^2} \to \cos^3 = \frac{1}{8} \to \cos = \frac{1}{2}$ $\left\{ 2k\pi + \frac{\pi}{3},\ 2k\pi + \frac{5\pi}{3} \right.$ $\frac{\pi}{3}$ ۲ جواب در بازه‌ی $[0, 2\pi]$ دارد $\frac{5\pi}{3}$ 1,75	۱
$5(1-\cos^2) + 2(4\cos^3 - 3\cos) = 0$ $5 - 5\cos^2 + 8\cos^3 - 6\cos = -2$ $8\cos^3 - 5\cos^2 - 6\cos + 7 = 0 \xrightarrow{y=[-1,1]} 8y^3 - 5y^2 - 6y + 7 = 0 \to (y+1)(8y^2 - 13y + 7)$ در کل ۴ جواب $\cos x = -1 \Rightarrow x = \pi$ $\cos x = 1 \Rightarrow x = 0$ $\cos x = \frac{10}{16} \xrightarrow{\text{دو جواب}}$ $y = \frac{13 \pm \sqrt{9}}{16}$ $\left\{ 1,\ \frac{10}{16} \right.$ 1	۲
$3\sin(1 - 2\sin^2) + \sin = 1$ $3\sin - 6\sin^3 + \sin = 1$ $-4\sin^2 + 4\sin - 1 = 0$ $-\frac{\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{\pi}{2}$ $\sin 3x - 1 = 0 \to \sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{\pi}{6} + \frac{2k\pi}{3}$ .75	۳
0	۴
$\cos x \times \frac{\sqrt{2}}{2} - \sin x \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{3}} \to \frac{\sqrt{2}}{2}(\cos - \sin) = \frac{1}{\sqrt{3}} \to \cos - \sin = \frac{2}{\sqrt{6}}$ 1 $m\left(\frac{2}{\sqrt{6}}\right) = 4\sqrt{6}\ \sin^{\frac{1}{2}} 2x \to m\left(\frac{2}{\sqrt{6}}\right) = \frac{4\sqrt{6}}{3} \to \boxed{m = 4}$ $\cos^2 + \sin^2 = \frac{4}{6}$ $1 - \sin 2x$ $\sin 2x = \frac{1}{3}$	۵

$x+\frac{\pi}{۶}-x+\frac{\pi}{۳}=\frac{\pi}{۲}$ ۲ $\sin(x+\frac{\pi}{۶})=۱ \begin{cases} x+\frac{\pi}{۶}=۲k\pi+\frac{\pi}{۲} \\ x+\frac{\pi}{۶}=۲k\pi-\frac{\pi}{۲} \end{cases}$	.۱۵	۶
$\sqrt{۳+۱}\ \sin(x+\frac{\pi}{۳})=\sqrt{۲} \to x+\frac{\pi}{۳}=۲k\pi+\frac{\pi}{۴}$ $\tan\alpha=\sqrt{۳}$ $\quad x+\frac{\pi}{۳}=۲k\pi+\frac{۳\pi}{۴}$	۰	۷
$۴\sin\cos=۱ \to \sin۲x=\frac{۱}{۲} \to ۲x=۲k\pi+\frac{\pi}{۶}$ $۲x=۲k\pi+\frac{۵\pi}{۶}$	۰	۸
	۰	۹
$\frac{\sin۳x}{\cos۳x}\neq\frac{\cos}{\sin} \to \sin۳x\sin x=\cos x\cos۳x \to \cos۴x=۰$ $۴x=۲k\pi+\frac{\pi}{۲}$ $۴x=۲k\pi-\frac{\pi}{۲}$	۰	۱۰

۲ -

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{-\pi \le x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک دو مجموعه جواب $\{\pi, -\pi\}$ است

۳

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow \underbrace{3\sin x - 4\sin^3 x}_{\sin 3x} = 1$

$\sin 3x = 1 \rightarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k = 0 \rightarrow x = \frac{\pi}{6}$, $k = 1 \rightarrow x = \frac{5\pi}{6}$, $k = 2 \rightarrow x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

۴

$\begin{cases} \sin\left(2x + \frac{\pi}{2}\right) = \cos 2x \\ \cos\left(\frac{\pi}{2} + 4x\right) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$

$\cos 2x - 2\sin 2x\cos 2x = 0 \rightarrow \cos 2x(1 - 2\sin 2x) = 0$ → $\cos 2x = 0$ ✗ ، $\sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$ یا $k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan\left(\frac{2\pi}{3}\right) = \boxed{-\sqrt{3}}$

۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{\sqrt{2}}{2}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

۶

$$\cos\left(x-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}-x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3}-x\right) = \sin\left(x+\frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right) = 1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right) = \pm 1 \longrightarrow x+\frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \to x = \frac{\pi}{3}, \quad k=1, x = \frac{4\pi}{3}$$

$\boxed{\text{معادله ۲ جواب در بازه دارد}}$

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

۹

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2 \alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \frac{\pm\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

$\star$ A نمی‌تواند برابر خود π باشد چون طبق $\star$ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!

۸

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \longrightarrow -2(2\sin x\cos x) = 1 \longrightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \text{ یا } 2k\pi + \frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi - \frac{\pi}{12}} \text{ یا } \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0 \le x \le 2\pi} \begin{cases} \overset{1}{} \; k = 0, 1 \longrightarrow x = \dfrac{7\pi}{12}, \dfrac{19\pi}{12} \\ \underset{2}{} \; k = 1, 2 \longrightarrow x = \dfrac{11\pi}{12}, \dfrac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

۱۰

$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{4\pi}$$