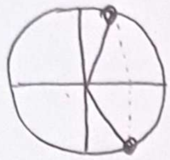
نام و نام خانوادگی ......امیرمحمد نادران‌جمی...... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس ...دوازدهم پسر B...

۱

$8\cos x - \tan^2 x = 1 \longrightarrow 8\cos x = 1 + \tan^2 x = \frac{1}{\cos^2 x} \longrightarrow 8\cos^3 x = 1$

$\cos^3 x = \frac{1}{8} \longrightarrow \cos x = \frac{1}{2}$



در بازه $[0, 2\pi]$

$\boxed{\text{۲ جواب}}$

۲

$5\sin^2 x + 2\cos(3x) = -2 \longrightarrow 5(1-\cos^2(x)) = 5 - 5\cos^2(x)$

$-5\cos^2 x + 2\cos(3x) = -7 \longrightarrow -5\cos^2 x + 2(\cos^4 x - 3\cos x) = -7$

$2\cos^4 x - 5\cos^2 x - 6\cos x = -7 \longrightarrow \cos x(2\cos^3 x - 5\cos x - 6) = -7$

۳

$2\sin(x)\cos(2x) + \sin(x) = 1$

۴

$\frac{1}{\sin(\frac{\pi+4x}{2})} + \frac{1}{\cos(\frac{\pi+8x}{2})} = 0 \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = \frac{1}{\cos 2x}\left(1 + \frac{1}{-2\sin 2x}\right) = 0$

نمی‌توانه صفر بشه

$1 + \frac{1}{-2\sin 2x} = -2\sin 2x + 1 = 0 \qquad 2\sin 2x = 1 \longrightarrow \sin 2x = \frac{1}{2}$

$2x = \frac{\pi}{6} \longrightarrow x = \frac{\pi}{12}$

$2x = \frac{5\pi}{6} \longrightarrow x = \frac{5\pi}{12}$

$\frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} = \alpha \qquad \tan 2\alpha = \tan\frac{2\pi}{3} = \boxed{-\sqrt{3}}$

۵

$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)=1 \longrightarrow x+\frac{\pi}{6}+\left(-x+\frac{\pi}{3}\right)=\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}$

$\sin\left(x+\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right) \longrightarrow \sin^2\left(x+\frac{\pi}{6}\right)=1 \begin{cases} \sin\left(x+\frac{\pi}{6}\right)=1 \rightarrow x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2} \\ \sin\left(x+\frac{\pi}{6}\right)=-1 \rightarrow x+\frac{\pi}{6}=2k\pi-\frac{\pi}{2} \end{cases}$ ۶

$\longrightarrow x=2k\pi+\frac{\pi}{3}$

$\longrightarrow x=2k\pi-\frac{2\pi}{3}$

$\boxed{\text{۲ جواب}}$

$\sin x+\sqrt{3}\cos x=\sqrt{2} \xrightarrow{\text{مطابق با}} a\sin x+b\cos x=\frac{|a|}{a}\times\sqrt{a^2+b^2}\times\sin(x+\alpha)\ ,\ \tan\alpha=\frac{b}{a}$

$\sin x+\sqrt{3}\cos x=\sqrt{4}\times\sin\left(x+\frac{\pi}{3}\right)=\sqrt{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$

$\sin\left(x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}\right) \begin{cases} x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \rightarrow x=2k\pi-\frac{\pi}{12} \longrightarrow -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4} \rightarrow x=2k\pi+\frac{5\pi}{12} \longrightarrow \frac{5\pi}{12} \end{cases}$ ۷

$\frac{(23+5-1)\pi}{12}=\frac{27\pi}{12}=\boxed{\frac{9\pi}{4}} \longrightarrow$ مجموع جواب‌ها

$4\sin x\ \sin\left(\frac{3\pi}{2}-x\right)=+1 \longrightarrow (4)(\sin x)(\cos x)(-1)=1$

$2\sin x\cos x\,(-2)=1 \longrightarrow \sin 2x=-\frac{1}{2} \begin{cases} 2x=2k\pi+\frac{7\pi}{6} \rightarrow x=k\pi+\frac{7\pi}{12} \\ 2x=2k\pi+\pi-\frac{7\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12} \end{cases}$ ۸

جواب‌ها $\longrightarrow \frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{(7+19+11+23)\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$

$(1+\cos(2\alpha))(1+\cos(4\alpha))(1+\cos(8\alpha))=\frac{1}{8} \rightarrow (\sin^2\alpha+\cos^2\alpha+\cos^2\alpha-\sin^2\alpha)=2\cos^2\alpha$

$=(2\cos^2(\alpha))(2\cos^2(2\alpha))(2\cos^2(4\alpha))=8\times(\cos^2\alpha\times\cos^2 2\alpha\times\cos^2 4\alpha)=\frac{1}{64}$ ۹

$|\cos\alpha\times\cos 2\alpha\times\cos 4\alpha|=\frac{1}{8} \xrightarrow{\times\sin x} \frac{1}{8}\sin 8x=\frac{\sin x}{8} \rightarrow \sin 8x=\sin x$

$8x=2k\pi+x \rightarrow x=\frac{2k\pi}{7}$ $\quad 8x=2k\pi+\pi-x \rightarrow x=\frac{2k\pi}{9}+\frac{\pi}{9} \rightarrow \boxed{Max=\frac{\pi}{2}}$

$\tan(3x)\tan(x)=1 \longrightarrow \tan(x)=\cot(3x)=\tan\left(\frac{\pi}{2}-3x\right)$

$\tan(x)=\tan\left(\frac{\pi}{2}-3x\right) \longrightarrow x=k\pi+\frac{\pi}{2}-3x \rightarrow 4x=k\pi+\frac{\pi}{2}$

$x=\frac{k\pi}{4}+\frac{\pi}{8}$ $\xrightarrow{\text{جواب‌ها در بازه } [\pi, 2\pi]}$ $\frac{4\pi}{4}+\frac{\pi}{8}, \frac{5\pi}{4}+\frac{\pi}{8}, \frac{6\pi}{4}+\frac{\pi}{8}, \frac{7\pi}{4}+\frac{\pi}{8}$ ۱۰

مجموع $\longrightarrow 4\times\frac{\pi}{8}+\frac{(4+5+6+7)\pi}{4}=\boxed{6\pi}$