۱۷٫۵

نام و نام خانوادگی ......علی درخشان فر...... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس ......B......

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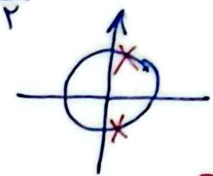
(۱) $8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1+\tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x}$

(۲) $8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$



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دوجواب

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(۱) $5(1-\cos^2 x)+2(4\cos^3 x-3\cos x)+2=0$

(۲) $8\cos^3 x-5\cos^2 x-6\cos x+7=0 \xrightarrow{\cos x=-1}$ (۳)

(۳): $(\cos x+1)(\underbrace{8\cos^2 x-13\cos x+7}) \Rightarrow \cos x=-1$

$\Delta<0 \Rightarrow$ ریشه ندارد

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دوجواب

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(۱) $2\sin x \times (1-2\sin^2 x)+\sin x=1 \rightarrow -4\sin^3 x+3\sin x-1=0$

(۲) $(\sin x+1)(-4\sin^2 x+4\sin x-1)=0 \Rightarrow \begin{cases} \sin x=-1 \Rightarrow x=\frac{3\pi}{2} \\ \sin x=\frac{1}{2} \Rightarrow x=\frac{\pi}{6},\frac{5\pi}{6} \end{cases}$

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(۳) $\frac{\pi}{6}+\frac{5\pi}{6}+\frac{3\pi}{2}=\frac{5\pi}{2}$

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(۱) $\frac{1}{\sin(\frac{\pi}{2}+2x)}+\frac{1}{\cos(\frac{\pi}{2}+4x)}=0 \Rightarrow \frac{1}{\cos 2x}-\frac{1}{\sin 4x}=0$

(۲) $\cos 2x=\sin 4x \Rightarrow$ $6x=90 \Rightarrow x=15^\circ$ ، $6x=270 \Rightarrow x=45$

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(۳) $\alpha=30^\circ \Rightarrow \boxed{\tan 2\alpha=\sqrt{3}}$

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(۱) $\cos(\frac{\pi}{4}+x)=\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x=\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}} \Rightarrow \cos x-\sin x=\frac{2}{\sqrt{6}}$

(۲) $\cos(x+\frac{\pi}{4})=\sin(\underbrace{\frac{\pi}{4}-x}_{t}) \longrightarrow \sin t=\frac{1}{\sqrt{3}} \Rightarrow \cos t=\sqrt{\frac{2}{3}}$

(۳) $\sin 2x=\cos(\underbrace{\frac{\pi}{2}-2x}_{2t}) \longrightarrow \cos 2t=\cos^2 t-\sin^2 t=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$

جایگذاری

(۴) $m(\frac{2}{\sqrt{6}})-3\sqrt{6}\times\frac{1}{3}=\sqrt{6} \Rightarrow m\frac{2}{\sqrt{6}}=2\sqrt{6} \Rightarrow \boxed{m=6}$

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گروه B — علی درخشان فر

① $\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3})=1 \xrightarrow{\cos x=\cos(-x)} \sin(x+\frac{\pi}{6})\cos(\frac{\pi}{3}-x)=1$

② $(x+\frac{\pi}{6}),(\frac{\pi}{3}-x) \xrightarrow{\text{متمم اند}} \sin^2(x+\frac{\pi}{6})=1$ ۶

$\sin(x+\frac{\pi}{6})=1 \Rightarrow x=\frac{\pi}{3}$

$\sin(x+\frac{\pi}{6})=-1 \Rightarrow x=\frac{4\pi}{3}$

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۲جواب

①' $\tan\alpha=\frac{b}{a}=\sqrt{3}\Rightarrow \alpha=\frac{\pi}{3}$

① $\sin x+\sqrt{3}\cos x=2\sin(x+\alpha)=2\sin(x+\frac{\pi}{3})=\sqrt{2}$

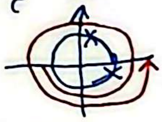
② $\sin(x+\frac{\pi}{3})=\sin(\frac{\pi}{4})$ ۷

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\to x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4}\to x=2k\pi+\frac{5\pi}{12}$

جوجواب ها ؟

$\boxed{\text{۳جواب}}$



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① $4\sin x\times(-\cos x)=-2\sin 2x=1$

$0\le x\le 2\pi$

① $\sin 2x=-\frac{1}{2}$

$2x=2k\pi+\frac{7\pi}{6}\Rightarrow x=k\pi+\frac{7\pi}{12}$

$2x=2k\pi+\pi-\frac{7\pi}{6}\Rightarrow x=k\pi-\frac{\pi}{12}$ ۸

$\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{23\pi}{12}+\frac{11\pi}{12}=\frac{60\pi}{12}=5\pi$

۱,۵

③ $x: \frac{7\pi}{12},\frac{19\pi}{12},\cancel{\frac{5\pi}{12}},\frac{11\pi}{12} \longrightarrow$ مجموع $=\frac{42\pi}{12}=\frac{7}{2}\pi$

① $\xrightarrow{1+\cos 2x=2\cos^2 x} (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$

② $\cos\alpha\times\cos 2\alpha\times\cos 4\alpha=\pm\frac{1}{8}\xrightarrow{\times\sin\alpha}\frac{\sin 8\alpha}{8}=\pm\frac{1}{8}\sin\alpha$ ۹

③ $\sin 8\alpha=\sin\alpha\longrightarrow\begin{cases}\alpha=\frac{2}{7}k\pi\to\max=\frac{6}{7}\pi\\ \alpha=\frac{2}{9}k\pi+\frac{\pi}{9}\to\max=\pi\end{cases}$

$\sin 8\alpha=\sin(-\alpha)\to\begin{cases}\alpha=\frac{2k\pi}{9}\to\max=\frac{8}{9}\pi\\ \alpha=\frac{2}{7}k\pi+\frac{\pi}{7}\to\max=\pi\end{cases}$

$\Rightarrow \boxed{\max=\pi}$

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* α نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

① $\tan 2x=\cot x\longrightarrow \tan 2x=\tan(\frac{\pi}{2}-x)$

② $3x=k\pi+\frac{\pi}{2}-x\longrightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$ ۱۰

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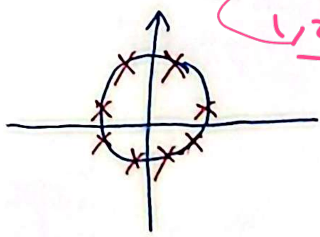

جوجواب ها ؟

۴جواب

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$$\begin{cases}\sin(2x+\frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2}+4x) = -\sin 4x\end{cases} \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \longrightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$$\cos 2x - 2\sin 2x\cos 2x = 0 \longrightarrow \cos 2x(1-2\sin 2x) = 0 \to \begin{cases}\cos 2x = 0 \ \times \\ \sin 2x = \frac{1}{2} \ \checkmark\end{cases}$$

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \ \text{یا}\ 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{\pi}{12} \ \text{یا}\ k\pi + \frac{5\pi}{12}$$

$$\xrightarrow{0 \le x \le \pi} k=0 \to x = \frac{\pi}{12} \ \text{یا}\ \frac{5\pi}{12} \to \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3} \to \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$$

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12}\end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

۸

$$\sin(\frac{3\pi}{2} - x) = -\cos x$$

$$4\sin x(-\cos x) = 1 \to -2(2\sin x\cos x) = 1 \to \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \ \text{یا}\ 2k\pi + \frac{7\pi}{6} \to x = \overset{2}{k\pi - \frac{\pi}{12}} \ \text{یا}\ \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0 \le x \le 2\pi} \begin{cases} 1: \ k = 0,1 \to x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ 2: \ k = 1,2 \to x = \frac{11\pi}{12}, \frac{23\pi}{12}\end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

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$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha\ *}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

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$$\tan 3x = \frac{1}{\tan x} = \cot x \rightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$