مرحله سازی - دوازدهم ریاضی B   به نام خدا   تکلیف ۱۷

۱۹٫۲۵ آفرین

۱- $8\cos\alpha - \sin^2\alpha - 1 \to 8\cos\alpha \cdot 1 + \sin^2\alpha \to 8\cos\alpha\cos^2\alpha \to 8\cos^3\alpha = 1 \to \cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{5\pi}{3} \to$ ۲ تا

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۲- $5\sin^2\alpha + 2\cos3\alpha = -2 \to 5 - 5\cos^2\alpha + 8\cos^3\alpha - 6\cos\alpha + 2 = 0$

$(\cos3\alpha = 4\cos^3\alpha - 3\cos\alpha)$ $\qquad 8\cos^3\alpha - 5\cos^2\alpha - 6\cos\alpha + 7 = 0$

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$(\cos\alpha + 1)(8\cos^2\alpha - 13\cos\alpha + 7) = 0 \qquad \Delta < 0 , \cos\alpha = -1 \to \alpha = \pi, -\pi$

جواب ندارد

۳- $2\sin\alpha\cos2\alpha + \sin\alpha = 1 \to 2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1 \quad (\cos2\alpha = 1 - 2\sin^2\alpha)$

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$(\sin\alpha + 1)(2\sin\alpha - 1)^2 = 0 \to \sin\alpha = -1 , \sin\alpha = \frac{1}{2} \qquad \sin\alpha = -1 \to \alpha = \frac{3\pi}{2}$

$(\frac{5\pi}{6} + \frac{\pi}{6}) + \frac{3\pi}{2} = \pi + \frac{3\pi}{2} = \frac{5\pi}{2}$

$\sin\alpha = \frac{1}{2} \to \alpha = \frac{\pi}{6} , \frac{5\pi}{6} \to \frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{2}$ دقت کن! ×۲

۴- $\sin(\frac{\pi}{2} + 2\alpha)^{-1} + \cos(\frac{\pi}{2} + 4\alpha)^{-1} \to \cos2\alpha = \sin4\alpha \to \cos2\alpha = 2\sin2\alpha\cos2\alpha$

$1 = 2\sin2\alpha \to \sin2\alpha = \frac{1}{2} \to 2\alpha = 2k\pi + \frac{\pi}{6} \to \alpha = k\pi + \frac{\pi}{12}$

$2\alpha = 2k\pi - \frac{\pi}{6} \to \alpha = k\pi + \frac{5\pi}{12}$

$\left\{ \alpha = \frac{5\pi}{12} , \frac{\pi}{12} , \frac{6\pi}{12} , \frac{\pi}{3} \right.$

$\sin2\alpha = \frac{2\pi}{3} = -\sqrt{3}$

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۵- $\cos(\alpha + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \to \cos^2(\alpha + \frac{\pi}{4}) = \frac{1}{3} \to -\sin2\alpha = -\frac{1}{3} \to \sin2\alpha = \frac{1}{3}$

$(\cos^2(\alpha + \frac{\pi}{4}) = \frac{1 + \cos(2\alpha + \frac{\pi}{2})}{2}) \qquad m(-\sqrt{2}\sin(\alpha - \frac{\pi}{4})) = \frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6}$

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$m = \frac{2\sqrt{6}}{\frac{\sqrt{6}}{3}} = 6$

مرحله ۴ ساری

۶- $\sin(\alpha+\frac{\pi}{6})\cos(\alpha-\frac{\pi}{3})=1 \to \cos(\frac{\pi}{3}-\alpha)\cos(\alpha-\frac{\pi}{3}) \to \sin^2(\alpha+\frac{\pi}{6})=1$

$\sin(\alpha+\frac{\pi}{6})=\pm1 \to \alpha+\frac{\pi}{6}=k\pi+\frac{\pi}{2} \to \alpha=k\pi+\frac{\pi}{3}$ جواب ۲

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۷- $\sin\alpha+\sqrt{3}\cos\alpha=\sqrt{2} \xrightarrow{\times\frac{1}{2}} \sin(\alpha+\frac{\pi}{3})=\frac{\sqrt{2}}{2} \to \alpha+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \to \alpha=2k\pi-\frac{\pi}{12}$

$\alpha+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4}$

$\alpha=2k\pi+\frac{5\pi}{12}$

$\alpha=\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{23\pi}{12} \to \frac{5\pi}{12}-\frac{\pi}{12}+\frac{23\pi}{12}=\frac{27\pi}{12}$

۸- $4\sin\alpha\sin(\frac{3\pi}{2}-\alpha)=1 \to 4\sin\alpha(-\cos\alpha)=1 \to -4\sin\alpha\cos\alpha=1$

$\sin2\alpha=-\frac{1}{2} \to 2\alpha=2k\pi-\frac{\pi}{6} \to \alpha=k\pi-\frac{\pi}{12}$, $2\alpha=2k\pi-\frac{5\pi}{6} \to \alpha=k\pi-\frac{5\pi}{12}$

$\to \frac{7\pi}{12}+\frac{11\pi}{12}+\frac{19\pi}{12}+\frac{23\pi}{12}=5\pi$

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۹- $2\cos^2\alpha=1+\cos2\alpha$ $\quad (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$

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$\cos\alpha\cdot\cos2\alpha\cdot\cos4\alpha=\pm\frac{1}{8} \xrightarrow{8\sin\alpha} \frac{\sin8\alpha}{8}=\pm\frac{1}{8}\sin\alpha$

$\sin8\alpha=\sin\alpha \to \alpha=\frac{2k\pi}{7}=\frac{6\pi}{7}$

$\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}=\pi$

$\sin8\alpha=\sin(-\alpha) \to \alpha=\frac{2k\pi}{9}=\frac{8\pi}{9}$

$\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}=\pi$

$\max\alpha=\pi$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

۱۰- $\tan3\alpha\tan\alpha=1 \to \tan3\alpha=\frac{1}{\tan\alpha}=\cot\alpha=\tan(\frac{\pi}{2}-\alpha) \to 3\alpha=k\pi+\frac{\pi}{2}-\alpha$

$4\alpha=k\pi+\frac{\pi}{2} \to \alpha=\frac{k\pi}{4}+\frac{\pi}{8} \to \frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=6\pi$

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$1+\cos 2x = 2\cos^2 x$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \bigstar}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا} \ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا} \ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \longrightarrow x = \frac{5\pi}{7} \qquad \longrightarrow \boxed{\max A = \frac{8\pi}{9}}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{7\pi}{9}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!