دوازدهم ریاضی B   به نام خدا   تکلیف ۱۷

۱-

$8\cos\alpha - \sin^2\alpha - 1 \to 8\cos\alpha \cdot 1 + \sin^2\alpha \to 8\cos\alpha \frac{1}{\cos^2\alpha} \to 8\cos^3\alpha = 1 \to \cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{5\pi}{3} \to$ ۲ تا

۲-

$5\sin^2\alpha + 2\cos 3\alpha = -2 \to 5 - 5\cos^2\alpha + 8\cos^3\alpha - 6\cos\alpha + 2 = 0$

$(\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha)$ $\quad 8\cos^3\alpha - 5\cos^2\alpha - 6\cos\alpha + 7 = 0$

$(\cos\alpha + 1)(8\cos^2\alpha + 13\cos\alpha + 7) = 0 \quad \underset{\text{جواب ندارد}}{\Delta < 0}, \cos\alpha = -1 \to \alpha = \pi, -\pi$

۳-

$2\sin\alpha\cos 2\alpha + \sin\alpha = 1 \to 2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1 \quad (\cos 2\alpha = 1 - 2\sin^2\alpha)$

$(\sin\alpha + 1)(2\sin\alpha - 1)^2 = 0 \to \sin\alpha = -1, \sin\alpha = \frac{1}{2} \quad \sin\alpha = -1 \to \alpha = \frac{3\pi}{2}$

$\sin\alpha = \frac{1}{2} \to \alpha = \frac{\pi}{6}, \frac{5\pi}{6} \to \frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{6}$

۴-

$\sin(\frac{\pi}{2} + 2\alpha)^{-1} + \cos(\frac{\pi}{2} + 4\alpha)^{-1} \to \cos 2\alpha = \sin 4\alpha \to \cos 2\alpha = 2\sin 2\alpha\cos 2\alpha$

$1 = 2\sin 2\alpha \to \sin 2\alpha = \frac{1}{2} \to 2\alpha = 2k\pi + \frac{\pi}{6} \to \alpha = k\pi + \frac{\pi}{12}$

$\qquad 2\alpha = 2k\pi - \frac{\pi}{6} \to \alpha = k\pi + \frac{5\pi}{12}$ $\quad \Big\}\ \alpha = \frac{5\pi}{12}, \frac{\pi}{12}, \frac{4\pi}{12}, \frac{\pi}{3}$

$\sin 2\alpha = \frac{2\pi}{3} = -\sqrt{3}$

۵-

$\cos(\alpha + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \to \cos(\alpha + \frac{\pi}{4})^2 = \frac{1}{3} \to -\sin 2\alpha = -\frac{1}{3} \to \sin 2\alpha = \frac{1}{3}$

$\left(\cos^2(\alpha + \frac{\pi}{4}) = \frac{1 + \cos(2\alpha + \frac{\pi}{2})}{2}\right) \quad m\left(-\sqrt{2}\sin(\alpha - \frac{\pi}{4})\right) = \frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6}$

$m = \frac{2\sqrt{6}}{\frac{\sqrt{6}}{3}} = 6$

مرحله م سازی

-۶
$\sin(\alpha+\frac{\pi}{6})\cos(\alpha-\frac{\pi}{3})=1 \to \cos(\frac{\pi}{3}-\alpha)\cos(\alpha-\frac{\pi}{3}) \to \sin^2(\alpha+\frac{\pi}{6})=1$

$\sin(\alpha+\frac{\pi}{6})=\pm 1 \to \alpha+\frac{\pi}{6}=k\pi+\frac{\pi}{2} \to \alpha=k\pi+\frac{\pi}{3}$ ۲ جواب

-۷
$\sin\alpha+\sqrt{3}\cos\alpha=\sqrt{2} \xrightarrow{\times\frac{1}{2}} \sin(\alpha+\frac{\pi}{3})=\frac{\sqrt{2}}{2} \to \alpha+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \to \alpha=2k\pi-\frac{\pi}{12}$

$\alpha+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4}$

$\alpha=2k\pi+\frac{5\pi}{12}$

$\alpha=\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{23\pi}{12} \to \frac{5\pi}{12}-\frac{\pi}{12}+\frac{23\pi}{12}=\frac{27\pi}{12}$

-۸
$4\sin\alpha\sin(\frac{3\pi}{2}-\alpha)=1 \to 4\sin\alpha(-\cos\alpha)=1 \to -4\sin\alpha\cos\alpha=-1$

$\sin 2\alpha=-\frac{1}{2} \to 2\alpha=2k\pi-\frac{\pi}{6} \to \alpha=k\pi-\frac{\pi}{12}$, $2\alpha=2k\pi-\frac{5\pi}{6} \to \alpha=k\pi-\frac{5\pi}{12}$

$\to \frac{7\pi}{12}+\frac{11\pi}{12}+\frac{19\pi}{12}+\frac{23\pi}{12}=5\pi$

-۹
$2\cos^2\alpha=1+\cos 2\alpha \quad (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$

$\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha=\pm\frac{1}{8} \xrightarrow{\times 8\sin\alpha} \frac{\sin 8\alpha}{8}=\pm\frac{1}{8}\sin\alpha$

$\sin 8\alpha=\sin\alpha \to \alpha=\frac{2k\pi}{7}=\frac{6\pi}{7}$

$\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}=\pi$

$\sin 8\alpha=\sin(-\alpha) \to \alpha=\frac{2k\pi}{9}=\frac{8\pi}{9}$

$\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}=\pi$

$\max\alpha=\pi$

-۱۰
$\tan 3\alpha\tan\alpha=1 \to \tan 3\alpha=\frac{1}{\tan\alpha}=\cot\alpha=\tan(\frac{\pi}{2}-\alpha) \to 3\alpha=k\pi+\frac{\pi}{2}-\alpha$

$4\alpha=k\pi+\frac{\pi}{2} \to \alpha=\frac{k\pi}{4}+\frac{\pi}{8} \to \frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=6\pi$