نام و نام خانوادگی .امیرعلی ... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس ...دهم...B...

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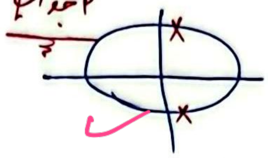
$8\cos x = \frac{1}{\cos^2 x} \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$

۲ جواب



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$7\sin^2 x + 2\cos^2 x + 2\cos(3x) = 0$

$7 - 7\cos^2 x$ ، $4\cos^3 x - 3\cos x$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \xrightarrow{\text{جمع ضرایب توان های زوج با جمع ضرایب توان های فرد برابر است}} (\cos x + 1)(8\cos^2 x - 13\cos x + 7)$

$\cos x = -1 \Rightarrow x = 2k\pi + \pi$

$x = -\pi , \pi$

۲ جواب ، $\Delta < 0$

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۳

$-4\sin^3 x + 3\sin x - 1 = 0 \Rightarrow (\sin x + 1)(-4\sin^2 x + 4\sin x - 1) = 0$

$-(2\sin x - 1)^2$

$\sin x = -1 \Rightarrow \sin x = \frac{3\pi}{2}$

$\sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6}$

$2k\pi + \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{6}$

$\frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$

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$\frac{1}{\cos 3x} - \frac{1}{\cos 5x} = 0 \Rightarrow \cos 3x = \cos 5x \Rightarrow 3x = 2k\pi \pm 5x$

$x = k\pi \Rightarrow x = \pi$

$x = \frac{k\pi}{4} \Rightarrow x = \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}$

$2x = 2\pi \Rightarrow \tan 4\pi = 0$

صورت سوال را با دقت بخوان :)

۱/۲۵

۵

$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \frac{\sqrt{6}}{3}$

$(\frac{\sqrt{6}}{3})^2 = 1 - 2\sin x\cos x$ ، $\frac{1}{3}$

$m \times \frac{\sqrt{6}}{3} - \sqrt{6} = \sqrt{6} \Rightarrow m = 6$

۲

٦

$\sin(x+\frac{\pi}{4})\cos(\frac{\pi}{4}-x)=1 \Rightarrow \frac{1}{2}\sin(2x+\frac{\pi}{2})=1$

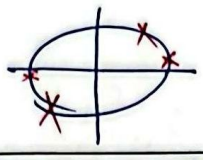
$\cos(x+\frac{\pi}{4})$

$\sin(2x+\frac{\pi}{2})=\sin\frac{\pi}{4}$ (1,5)

$2x+\frac{\pi}{2}=2k\pi+\frac{\pi}{4} \Rightarrow x=k\pi-\frac{\pi}{8}$

$2x+\frac{\pi}{2}=2k\pi+\pi-\frac{\pi}{4} \Rightarrow x=k\pi+\frac{\pi}{8}$

٥ حلول



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$a\sin x+b\cos x=\frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)$ $\quad \tan\alpha=\frac{b}{a}$

$\sin x+\sqrt{3}\cos x=2\sin(x+\frac{\pi}{3})=\sqrt{2}$

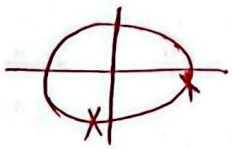
$\sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$

$\sin(x+\frac{\pi}{3})=\sin\frac{\pi}{4}$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \Rightarrow x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4} \Rightarrow x=2k\pi+\frac{5\pi}{12}$ (1,5)

$-\frac{7\pi}{12}-\frac{\pi}{12}+\frac{17\pi}{12}+\frac{23\pi}{12}$



$\frac{33\pi}{12}$

٨

$-2\sin 2x=1$

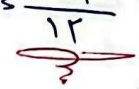
$\sin 2x=-\frac{1}{2}$

$2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12}$

$2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12}$ $\rightarrow k=2 \rightarrow x=2\pi-\frac{\pi}{12}$

$x=\frac{23\pi}{12}$

$\frac{11\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\frac{37\pi}{12}$



(1,75)

$\frac{1}{4}\sin^2 4\alpha \quad \frac{1}{16}\sin^2 4\alpha \quad \frac{1}{64}\sin^2 8\alpha$

$S=\frac{11\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{23\pi}{12}=\frac{4\pi}{3}=5\pi$

٩

$\sin^2\alpha\times(\cos^2\alpha)(\cos^2 2\alpha)(\cos^2 4\alpha)=\frac{1}{8}$

$\sin^2\alpha$

$\Rightarrow 2\sin^2 8\alpha=8\sin^2\alpha$

$8\sin 8\alpha=4\sqrt{2}\sin\alpha$ (1)

١٠

$\tan(2x)=\cot x \Rightarrow 2x=\frac{\pi}{2}-x+k\pi \Rightarrow x=\frac{k\pi}{3}+\frac{\pi}{6}$

$\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}$ ✓

(2)

(۴)

$$\begin{cases}\sin(2\alpha+\frac{\pi}{2})=\cos 2\alpha\\ \cos(\frac{\pi}{2}+\beta)=-\sin\beta\end{cases}\xrightarrow{\alpha=\beta=2\alpha}\frac{1}{\cos 2\alpha}+\frac{1}{-\sin 4\alpha}=0\rightarrow\frac{\cos 2\alpha-\sin 4\alpha}{\cos 2\alpha(-\sin 4\alpha)}=0$$

$$\cos 2\alpha-2\sin 2\alpha\cos 2\alpha=0\rightarrow\cos 2\alpha(1-2\sin 2\alpha)=0$$

$\cos 2\alpha=0$ ✗

$\sin 2\alpha=\frac{1}{2}$ ✓

* اگر $\cos 2\alpha=0$ باشد مضرب عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$$\sin 2\alpha=\frac{1}{2}\rightarrow 2\alpha=2k\pi+\frac{\pi}{6}\ \text{یا}\ 2k\pi+\frac{5\pi}{6}\rightarrow\alpha=k\pi+\frac{\pi}{12}\ \text{یا}\ k\pi+\frac{5\pi}{12}$$

$$0\le\alpha\le\pi\xrightarrow{}k=0\rightarrow\alpha=\frac{\pi}{12}\ \text{یا}\ \frac{5\pi}{12}\rightarrow\frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3}$$

$$\alpha=\frac{\pi}{3}\rightarrow\tan(2\alpha)=\tan(\frac{2\pi}{3})=\boxed{-\sqrt{3}}$$

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(۶)

$$\cos(\alpha-\frac{3\pi}{2})=\cos(\frac{3\pi}{2}-\alpha)$$

$$\frac{3\pi}{2}-\alpha+\alpha+\frac{\pi}{4}=\frac{7\pi}{4}\rightarrow\cos(\frac{3\pi}{2}-\alpha)=\sin(\alpha+\frac{\pi}{4})$$

$$\sin^2(\alpha+\frac{\pi}{4})=1\rightarrow\sin(\alpha+\frac{\pi}{4})=\pm1\rightarrow\alpha+\frac{\pi}{4}=k\pi+\frac{\pi}{2}$$

$$\alpha=k\pi+\frac{\pi}{4}\xrightarrow{0\le\alpha\le2\pi}k=0\rightarrow\alpha=\frac{\pi}{4}\ ,\ k=1\ ,\ \alpha=\frac{5\pi}{4}$$

معادله ۲ جواب در بازه دارد

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(۷) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin\alpha+\frac{\sqrt{3}}{2}\cos\alpha=\frac{\sqrt{2}}{2}$$

$$\cos 45^\circ\sin\alpha+\sin 45^\circ\cos\alpha=\frac{\sqrt{2}}{2}\rightarrow\sin(\alpha+\frac{\pi}{4})=\frac{\sqrt{2}}{2}$$

$$\begin{cases}\alpha+\frac{\pi}{4}=2k\pi+\frac{\pi}{4}\rightarrow\alpha=2k\pi\xrightarrow{0\le\alpha\le2\pi}k=0,1\rightarrow\alpha=0,2\pi\\ \alpha+\frac{\pi}{4}=2k\pi+\frac{3\pi}{4}\rightarrow\alpha=2k\pi+\frac{\pi}{2}\xrightarrow{0\le\alpha\le2\pi}k=0\rightarrow\alpha=\frac{\pi}{2}\end{cases}$$

$$S=\frac{2\pi}{12}-\frac{\pi}{12}+\frac{\pi}{2}=\frac{4\pi}{12}=\boxed{\frac{\pi}{3}}$$

(۹)

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} \cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} \text{(*)}$$

$$(\underbrace{\sin\alpha \cos\alpha}_{\frac{1}{2}\sin 2\alpha}) \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}(\underbrace{\sin 2\alpha \cos 2\alpha}_{\frac{1}{2}\sin 4\alpha}) \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}(\underbrace{\sin 4\alpha \cos 4\alpha}_{\frac{1}{2}\sin 8\alpha}) = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \quad \text{یا} \quad 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \quad \text{یا} \quad 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{MAn\,A} k=3 \rightarrow n = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{MAn\,A} k=4 \rightarrow n = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAn\,A} k=2 \rightarrow n = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAn\,A} k=3 \rightarrow n = \frac{7\pi}{9}$$

$$\rightarrow \boxed{MAn_A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π یا 2π چون طبق * $\sin A$ را مخالف صفر در نظر گرفته‌ایم!

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(۷)

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \rightarrow -2(2\sin x\cos x) = 1 \rightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \quad \text{یا} \quad 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12} \quad \text{یا} \quad k\pi + \frac{7\pi}{12}$$

$$0 \le x < 2\pi \rightarrow \begin{cases} k=0,1 \rightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ k=1,2 \rightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$