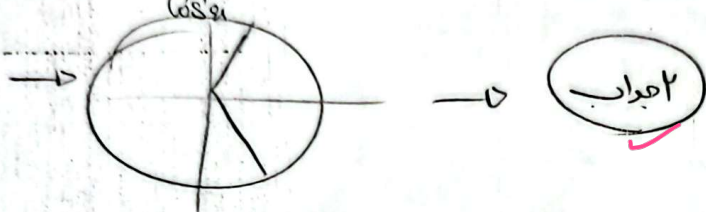


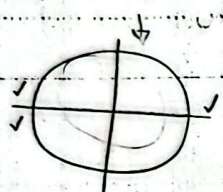
$$1 \cos \alpha = \frac{1}{\cos^2 \alpha} \rightarrow \cos \alpha \neq 0 \rightarrow 1 \cos^3 \alpha = 1 \rightarrow 2 \cos \alpha = 1 \rightarrow \cos \alpha = \frac{1}{2}$$



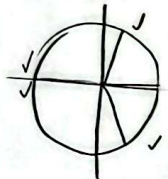
۲ جواب

این عبارت سه تار صورتی جواب دارد که $2 \cos(\alpha) = -2$ و $\sin^2 \alpha = 0$ باشد

$$\rightarrow \sin^2 \alpha = 0$$



$$\cos^2 \alpha = -1$$



نتیجه: ۱ و ۲ - مشترک = ۲ جواب

$$\sin \alpha (2 \cos(\alpha) + 1) = 0 \rightarrow \sin \alpha (4 \cos^2 \alpha - 1) = \sin \alpha (3 - 4 \sin^2 \alpha) = 1$$

$$4 \cos^2 \alpha = 2 \cos \alpha + 1$$

$$\rightarrow 3t - 4t^3 = 1 \rightarrow 4t^3 - 3t + 1 = 0 \rightarrow (t+1)(4t^2 - 4t + 1) = (t+1)(2t-1)^2 \rightarrow \sin \alpha = 1 \rightarrow \alpha = \frac{\pi}{2} + 2k\pi$$

$$\frac{1}{\sin(\frac{\pi}{2} + \alpha)} + \frac{1}{\cos(\frac{\pi}{2} + \alpha)} = 0 \rightarrow \frac{1}{\cos \alpha} - \frac{1}{\sin \alpha} = 0 \rightarrow \frac{1}{\cos \alpha} = \frac{1}{\sin \alpha} \rightarrow \tan \alpha = 1$$

$$\frac{2 \sin \alpha - 1}{\sin \alpha} = 0 \rightarrow 2 \sin \alpha = 1 \rightarrow \sin \alpha = \frac{1}{2}, \cos \alpha \neq 0 \rightarrow \alpha = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$\rightarrow \alpha = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} \rightarrow \tan \alpha = \tan \frac{2\pi}{3} = -\sqrt{3}$$

$$\cos(\alpha + \frac{\pi}{2}) = \cos \alpha \cos \frac{\pi}{2} - \sin \alpha \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} (\cos \alpha - \sin \alpha) = \frac{\sqrt{2}}{2} \rightarrow \cos \alpha - \sin \alpha = \frac{\sqrt{2}}{2}$$

$$\frac{m\sqrt{2}}{3} - 3\sqrt{2} \sin \alpha = \sqrt{2} \rightarrow \frac{m}{3} - 3 \sin \alpha = 1 \rightarrow (\cos \alpha \sin \alpha)^2 = 1 - \sin^2 \alpha = \frac{4}{9}$$

$$\rightarrow \sin \alpha = \frac{1}{3} \rightarrow \frac{m}{3} - 1 = 1 \rightarrow m = 4$$

$$\sin\left(\frac{\pi}{4} + \alpha - \frac{\pi}{4}\right) \cos\left(\alpha - \frac{\pi}{4}\right) = 1 \rightarrow \cos^2\left(\alpha - \frac{\pi}{4}\right) = 1$$

$$\rightarrow \cos\left(\alpha - \frac{\pi}{4}\right) = \pm 1 \rightarrow \text{Unit Circle} \rightarrow \alpha = \left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi, \frac{7\pi}{4}\right\} \Rightarrow \text{Answer}$$

$$\frac{1}{\sqrt{2}} \sin \alpha + \frac{\sqrt{2}}{2} \cos \alpha = \frac{\sqrt{2}}{2} \rightarrow \sin \alpha \cos \frac{\pi}{4} + \cos \alpha \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\left(\alpha - \frac{\pi}{4}\right)$$

$$\rightarrow \alpha - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{2} \rightarrow \begin{cases} \alpha = 2k\pi + \frac{\pi}{2} + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \rightarrow \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4} \\ \alpha = 2k\pi - \frac{\pi}{2} + \frac{\pi}{4} = 2k\pi - \frac{\pi}{4} \rightarrow \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4} \end{cases}$$

$$\frac{3\pi}{4} + 2\pi - \frac{7\pi}{4} = 2\pi + \frac{3\pi}{4}$$

$$-\sqrt{2} \sin \alpha \cos \alpha = 1 \rightarrow -\sqrt{2} \sin 2\alpha = 1 \rightarrow \sin 2\alpha = -\frac{1}{\sqrt{2}}$$

$$2\alpha = 2k\pi - \frac{\pi}{4} \rightarrow \alpha = k\pi - \frac{\pi}{8} \rightarrow \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}, \frac{19\pi}{8} = \frac{7\pi}{8}$$

$$2\alpha = 2k\pi - \frac{3\pi}{4} \rightarrow \alpha = k\pi - \frac{3\pi}{8} \rightarrow \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{17\pi}{8}$$

$$(1 - \cos 2\alpha)(1 + \cos 2\alpha) = 1 - \cos^2 2\alpha = 1 - \frac{\cos 4\alpha + 1}{2} = \frac{1 - \cos 4\alpha}{2}$$

$$\rightarrow A \times \frac{(1 - \cos 2\alpha)}{(1 - \cos 4\alpha)} = \frac{1}{A} \times \frac{(1 - \cos 4\alpha)}{(1 - \cos 4\alpha)} = \frac{1}{A} \rightarrow \cos 4\alpha = \cos 2\alpha \rightarrow \begin{cases} 4\alpha = 2k\pi + 2\alpha \\ 4\alpha = 2k\pi - 2\alpha \end{cases}$$

$$\rightarrow \alpha = \frac{2k\pi}{2}, \cos 2\alpha \neq 0 \rightarrow A = \frac{1}{2}$$

$$\tan 2\alpha = \tan\left(\frac{\pi}{4} - \alpha\right) \rightarrow a = k\pi + b \rightarrow 2\alpha = k\pi + \frac{\pi}{4} - \alpha \rightarrow 3\alpha = k\pi + \frac{\pi}{4} \rightarrow \alpha = \frac{k\pi}{3} + \frac{\pi}{12}$$

$$\rightarrow \frac{\pi}{12} + \frac{5\pi}{12} + \frac{9\pi}{12} + \frac{13\pi}{12} = \frac{28\pi}{12} = \frac{7\pi}{3}$$