۱۸٫۵

نام و نام خانوادگی ......امیرمحمد ابوالعالی

پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱	$\lambda \cos x - \tan^2 x = 1 \longrightarrow \lambda \cos x = 1 + \tan^2 \longrightarrow \lambda \cos x = \frac{1}{\cos^2 x}$ $\lambda \cos^3 x = 1 \longrightarrow \cos x = \frac{1}{2} \longrightarrow 2k\pi \pm \frac{\pi}{3}$ ۲ جواب ✓ ۲
۲	$5\sin^2 x + 2\cos(3x) = -2 \longrightarrow 5 - 5\cos^2 x + \lambda\cos^3 x - 6\cos x = -2$ $\underbrace{(1-\cos^2)}$ $\underbrace{4\cos^3 x - 3\cos x}$ $\lambda\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \longrightarrow (\cos\alpha + 1)(\lambda\cos^2\alpha - 13\cos\alpha + 7) = 0$ $\boxed{\cos\alpha = -1} \longrightarrow \alpha = 2k\pi + \pi \xrightarrow{k=-1} \alpha = -\pi$ یک جواب ✗ ۲ جواب ۱٫۷۵
۳	$2\sin(x)\cos(2x) + \sin x = 1 \longrightarrow \sin x(\underbrace{2\cos 2x + 1}) = 1$ $2\cos^2 x - 2\sin^2 x + \sin^2 + \cos^2$ $-\sin^2 x$ $\sin x(3\cos^2 x - \sin^2 x) = 1$ $(1-\sin^2 x)$ $\sin x(3 - 4\sin^2 x) = 1 \longrightarrow \underbrace{3\sin x - 4\sin^3 x}_{\sin(3x)} - 1 = 0 \longrightarrow \sin(3x) - 1 = 0$ ✓ $3x = 2k\pi + \frac{\pi}{2} \rightarrow \boxed{x = \frac{2k\pi}{3} + \frac{\pi}{6}} \sim \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \sim \pi + \frac{\pi}{2}$ $\frac{5\pi}{2}$ ۲
۴	$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \longrightarrow \frac{1}{\cos(2x)} + \frac{1}{\underbrace{-\sin(4x)}_{-2\sin(2x)\cos(2x)}} = 0$ $\frac{\cos 2x - 2\sin(2x)\cos(2x)}{-2\sin(2x)\cos(2x)^2} = 0 \longrightarrow \cos(2x) - 2\sin(2x)\cos(2x) = 0$ $\cos(2x)(1 - 2\sin(2x)) = 0 \longrightarrow \cos 2\alpha = 0 \longrightarrow$? ۱
۵	$\cos(x + \frac{\pi}{4}) = \frac{\sqrt{3}}{3} \sim \sin 2\alpha = \frac{2}{3}$ $m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6} \longrightarrow (m)(\sqrt{2}\cos(x + \frac{\pi}{4})) - 3\sqrt{6}\sin 2x$ $\frac{\sqrt{2}m}{\sqrt{3}} - 3\sqrt{6}\,\frac{2}{3} \longrightarrow \frac{\sqrt{2}}{\sqrt{3}} m - 2\sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} - 2\sqrt{6} = \sqrt{6}$ $3\sqrt{6} = m\sqrt{6} - 2\sqrt{6} \longrightarrow \boxed{m = 5}$ ۱٫۷۵

$\sin\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right)=1$

$\sin\left(x+\frac{\pi}{4}\right)\underbrace{\cos\left(\frac{\pi}{4}-x\right)}_{\sin\left(x+\frac{\pi}{4}\right)}=1$

$\sin^2\left(x+\frac{\pi}{4}\right)=1 \rightarrow \sin\left(x+\frac{\pi}{4}\right)=\pm 1 \rightarrow \boxed{\begin{matrix} x=2k\pi+\frac{\pi}{4} \\ x=2k\pi+\frac{4\pi}{3} \end{matrix}}$ ۲ ۲-۲ ۶

$\sin x+\sqrt{3}\cos x=\sqrt{3}$

$\sqrt{1+3}\sin(x+a) \rightarrow 2\sin\left(x+\frac{\pi}{3}\right)=\sqrt{3} \rightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

$\tan a=\sqrt{3} \rightarrow a=60^\circ=\frac{\pi}{3}$ ۷

$x=-\frac{\pi}{12},\ x=\frac{5\pi}{12},\ x=\frac{23\pi}{12} \rightarrow$

$\frac{9\pi}{4}$ ۲

$4\sin x\sin\left(\frac{3\pi}{2}-x\right)=1 \rightarrow -4\sin x\cos x=1 \rightarrow -2(2\sin x\cos x)=1$

$\rightarrow -2\sin(2\alpha)=1 \rightarrow \sin 2\alpha=-\frac{1}{2} \rightarrow \alpha=k\pi+\frac{7\pi}{12}$ و $k\pi+\frac{11\pi}{12}$

$\frac{7\pi}{12},\ \frac{11\pi}{12},\ \frac{19\pi}{12},\ \frac{23\pi}{12} \rightarrow \boxed{5\pi}$ ۸

۲

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\cancel{\frac{1}{16}}$

$\cancel{\cos^2\alpha\times\cos^2 2\alpha\times\cos^2 4\alpha=\frac{1}{16}}$

$(2\cos^2\alpha)\times(2\cos^2 2\alpha)\times(2\cos^2 4\alpha)=\frac{1}{8}\times\frac{4\sin^2\alpha}{4\sin^2\alpha} \rightarrow 4\sin^2\alpha\cos^2\alpha\times 2\cos^2 2\alpha\times 2\cos^2 4\alpha=\frac{4\sin^2\alpha}{8}$ ۹ ۲

$\rightarrow \frac{1}{4}\sin^2 8\alpha=\frac{\sin^2\alpha}{4} \rightarrow \sin^2 8\alpha=\sin^2\alpha \rightarrow 8\alpha=k\pi\pm\alpha$

$\frac{k\pi}{9},\ \frac{k\pi}{7}$ $\boxed{\max} \rightarrow \frac{8\pi}{9}$

$\tan(3\alpha)\tan(\alpha)=1 \rightarrow 1-\tan(3\alpha)\tan(\alpha)=0$

$\tan 4\alpha=\frac{\tan\alpha+\tan 3\alpha}{1-\tan 3\alpha\tan\alpha}$ تعریف نشده $\rightarrow \tan 4\alpha=\frac{\sin 4\alpha}{\cos 4\alpha}$ ۲

$\rightarrow \cos 4\alpha=0 \rightarrow 4\alpha=k\pi+\frac{\pi}{2} \rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$ ۱۰

$\frac{9\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}+\frac{11\pi}{8}=\frac{48\pi}{8}$ 6π

④

$$\begin{cases} \sin(2x+\frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2}+4x) = -\sin 4x \end{cases} \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$$\cos 2x - 2\sin 2x \cos 2x = 0 \longrightarrow \cos 2x(1 - 2\sin 2x) = 0 \begin{cases} \rightarrow \cos 2x = 0 \ \times \\ \rightarrow \sin 2x = \frac{1}{2} \ \checkmark \end{cases}$$

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \ \text{یا} \ 2k\pi + \frac{5\pi}{6} \longrightarrow x = k\pi + \frac{\pi}{12} \ \text{یا} \ k\pi + \frac{5\pi}{12}$$

$$\xrightarrow{0 \le x \le \pi} k = 0 \longrightarrow x = \frac{\pi}{12} \ \text{یا} \ \frac{5\pi}{12} \longrightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3} \longrightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$$

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⑤

$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m = 6}$$

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⑥ -

$$\cos 3x = 2\cos^2\frac{3x}{2} - 1 \quad (\text{فرمول طلایی})$$

$$5\sin^2 x + 2(2\cos^2\frac{3x}{2} - 1) = -2 \longrightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده‌اند!

$$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi \ \text{یا} \ -\pi \ \text{یا} \ 0$$

$$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$$

$$\xrightarrow{-\pi \le x \le \pi} x = -\pi \ \text{یا} \ -\frac{\pi}{3} \ \text{یا} \ \frac{\pi}{3} \ \text{یا} \ \pi$$

اشتراک دو مجموعه جواب $\{\pi, -\pi\}$ است