تکلیف ۷

①

$8\cos x - \tan^2 x = 1 \to 1+\tan^2 x = 8\cos x \to \frac{1}{\cos^2 x} = 8\cos x$

$\cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3}$, $\frac{\pi}{3}, \frac{5\pi}{3}$

$x \neq k\pi + \frac{\pi}{2}$

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②

$5\sin^2 x + 2\cos 3x = -2 \to 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$8t^3 - 5t^2 - 6t + 7 = 0$

$\cos x = -1 \leftarrow (t+1)(8t^2 - 13t + 7)$

$x \in \pi, -\pi$

۲ جواب

$\Delta = 169 - 224 < 0$ جواب ندارد

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③

$\sin x(2\cos 2x + 1) = 1$

$\sin x(2(1-2\sin^2 x)+1) = 1$

$\sin x(-4\sin^2 x + 3) = 1 \to \sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6} = \frac{(4k+1)\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}$

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④

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \to \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x}$

$\frac{2\sin 2x\cos 2x - \cos 2x}{2\cos 2x \sin 2x \cos 2x} = \frac{(2\sin 2x - 1)\cancel{\cos 2x}}{2\sin 2x\cos 2x\cancel{\cos 2x}} = 0 \to \sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6}$

$x = k\pi + \frac{\pi}{12}$

$x = 2k\pi + \frac{5\pi}{6}$

$x = k\pi + \frac{5\pi}{12}$

$\frac{5\pi}{12} - \frac{\pi}{12} = \alpha = \frac{4\pi}{12} = \frac{\pi}{3} \to \tan\frac{2\pi}{3} = -\sqrt{3}$

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⑤

$\cos(x+\frac{\pi}{4}) = \sin(\frac{\pi}{4}-x) = -\sin(x-\frac{\pi}{4}) = \frac{-\sqrt{2}}{\sqrt{2}}\sin(x-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}}$

$\cos x - \sin x = \sqrt{\frac{2}{3}}$

$m \times \frac{\cancel{\sqrt{2}}}{\sqrt{3}} - 3\sqrt{3}\times\cancel{\sqrt{2}}\sin 2x = \cancel{\sqrt{2}}\times\sqrt{3}$

$\cos^2 x + \sin^2 x - 2\sin x\cos x = \frac{2}{3}$

$\frac{\cancel{\sqrt{3}}}{3}m - 3\cancel{\sqrt{3}}\sin 2x = \cancel{\sqrt{3}}\,1 \xrightarrow{I} \frac{m}{3} - 1 = 1 \to m = 6$

$1 - \sin 2x = \frac{2}{3}$

$\sin 2x = \frac{1}{3}$ I

(6)

$$\underbrace{\sin\left(x+\frac{\pi}{6}\right)}_{\alpha}\underbrace{\cos\left(x-\frac{\pi}{3}\right)}_{\beta}=1$$

$$\alpha-\beta=\frac{\pi}{2}\to\alpha=\beta+\frac{\pi}{2}\to\sin\left(x-\frac{\pi}{3}+\frac{\pi}{2}\right)\cos\left(x-\frac{\pi}{3}\right)=1$$

$$\cos\left(x-\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=1\to\cos^2\left(x-\frac{\pi}{3}\right)=1\to\cos\left(x-\frac{\pi}{3}\right)=1$$

$$\cos\left(x-\frac{\pi}{3}\right)=-1$$

$$x-\frac{\pi}{3}=k\pi\to x=k\pi+\frac{\pi}{3}\to x=\frac{\pi}{3}, \frac{4\pi}{3}$$

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(7)

$$\sin x+\sqrt{3}\cos x=\sqrt{2}\to\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2}\to\sin\left(x+\frac{\pi}{3}\right)=\sin\frac{\pi}{4}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\to x=2k\pi+\frac{\pi}{4}-\frac{\pi}{3}=\frac{24k\pi-\pi}{12}=\frac{(24k-1)\pi}{12}\to\frac{-\pi}{12}, \frac{23\pi}{12}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4}\to x=2k\pi+\frac{3\pi}{4}-\frac{\pi}{3}=\frac{(24k+5)\pi}{12}\to\frac{5\pi}{12}$$

$$\frac{5\pi}{12}-\frac{\pi}{12}+\frac{23\pi}{12}=\frac{27\pi}{12}=\frac{9\pi}{4}$$

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(8)

$$4\sin x\sin\left(\frac{3\pi}{2}-x\right)=1\to-4\sin x\cos x=1\to\sin 2x=-\frac{1}{2}\to 2x=2k\pi-\frac{\pi}{6}\to x=k\pi-\frac{\pi}{12}$$

$$2x=2k\pi+\frac{7\pi}{6}\to x=k\pi+\frac{7\pi}{12}$$

$$\frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}\Rightarrow\frac{(\cancel{11}+\cancel{23}+\cancel{7}+19)\pi}{12}=5\pi$$

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(9)

$$(1+\cos(2\alpha))(1+\cos(4\alpha))(1+\cos(8\alpha))=\frac{1}{8}\to 2\cos^2\alpha\times 2\cos^2 2\alpha\times 2\cos^2 4\alpha=\frac{1}{8}$$

$$\cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}\longrightarrow\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}\sin\alpha$$

$$\to\frac{1}{2}\sin 2\alpha$$

$$\frac{1}{4}\sin 4\alpha$$

$$\frac{1}{\cancel{8}}\sin 8\alpha=\pm\frac{1}{\cancel{8}}\sin\alpha\to\sin 8\alpha=\pm\sin\alpha$$

$$8\alpha=k\pi\pm\alpha\to\alpha=\frac{k\pi}{7}\to\frac{4\pi}{7}\ max$$

$$\alpha=\frac{k\pi}{9}\to\frac{4\pi}{9}\ max\longrightarrow max$$

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(10)

$$\tan x\times\tan(3x)=1\longrightarrow\tan 3x=\cot x=\tan\left(\frac{\pi}{2}-x\right)$$

$$\tan x\times\cot x=1$$

$$3x=k\pi+\frac{\pi}{2}-x\to x=\frac{k\pi}{4}+\frac{\pi}{8}$$

$$\downarrow$$

$$\frac{(2k+1)\pi}{8}\qquad\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

$x\neq k\pi+\frac{\pi}{2}\to$ جواب‌های! اشتراک ندارد

$3x\neq k\pi+\frac{\pi}{2}\to x\neq\frac{k\pi}{3}+\frac{\pi}{6}\to\frac{(2k+1)\pi}{6}$ با جواب‌های اشتراک ندارد