اسر کاشف

۱)

الف) $\lim_{x\to 2^+} \frac{2x+5}{x+1} = \frac{4+5}{3} = 3$

ب) $\lim_{x\to 2^-} \frac{2x+5}{x+1} = \frac{4+5}{3} = 3$

ج) $\lim_{x\to 2} \frac{2x+5}{x+1} = 3$ ⇐ حد چپ و راست برابر ۳ پس حد خودش می‌شه ۳

د) $\lim_{x\to 0} \frac{2x+7}{[x^2]} = \frac{7}{[0^+]} = \frac{7}{0} =$ تعریف نشده

۲)

الف) $\lim_{x\to 3^-} 4[x]-2 = 4\times 2-2 = 6$

ب) $\lim_{x\to 3^-}[4x-2] = [10^-] = 9$

ج) $[\lim_{x\to 3^-} 4x-2] = [\lim 10] = 10$

د) $[\lim_{x\to 3^+} 4x-2] = 10$

۳)

الف) $\lim_{x\to 3^-} \frac{3[x]+1}{2} = \frac{3\times 2+1}{2} = 3{,}5$

ب) $\lim_{x\to 3^-} \left[\frac{3x+1}{2}\right] = \left[\frac{10^-}{2}\right] = [5^-] = 4$

ج) $\left[\lim_{x\to 3^-} \frac{3x+1}{2}\right] = \left[\frac{3\times 3+1}{2}\right] = 5$

د) $\left[\lim_{x\to 3^+} \frac{3x+1}{2}\right] = \left[\frac{3\times 3+1}{2}\right] = 5$

۴

الف) $\lim_{x\to 1} \frac{x-5}{x^2-6x+5} = \frac{\cancel{x-5}}{\cancel{(x-5)}(x-1)} = \frac{1}{x-1}$

$1^+ \to +\infty$

$1^- \to -\infty$

ب) $\lim_{x\to 3} \frac{x-4}{x^2-6x+9} = \frac{x-4}{(x-3)^2}$

$3^+ \to \frac{-1}{0^+} = -\infty$

$3^- \to \frac{-1}{0^+} = -\infty$

۵

الف) $\lim_{x\to -3} [-3x]-[5x]$

$-3^+ \to [9^-]-[-15^+] = 8+15 = 23$

$-3^- \to [9^+]-[-15^-] = 9-(-16) = 25$

ب) $\lim_{x\to (-\frac{1}{2})^-} [-\frac{1}{x^4}]$

$x < -\frac{1}{2} \Rightarrow \frac{1}{x} > -2 \Rightarrow \frac{1}{x^4} < 16 \Rightarrow -\frac{1}{x^4} > -16 \to [-\frac{1}{x^4}] = -16$

۶

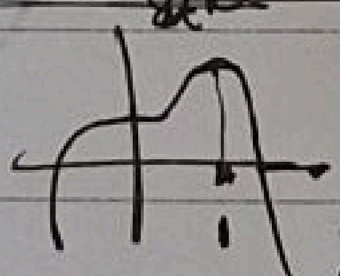
الف) $\lim_{x\to 1} [4x^3-3x^4+3] = [4-3+3] = 3$

$1^+ , 1^-$; 4^-

مشتق می‌گیرم و تابع را رسم می‌کنم

$12x^2-12x^3 = 12x^2(1-x)$

x		0		1	
f'	+		+		−
f	↗		↗		↘



ب) $\lim_{x\to 0} [4x^3-3x^4+3]$

$0^+ \to [3^-] = 2$

$0^- \to [3^+] = 3$

۷

الف) $\lim_{x\to 2} \frac{|x^3-8|}{x^2-4}$

$2^+ \to \frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)} = \frac{12}{4} = 3$

$2^- \to \frac{8-x^3}{x^2-4} = \frac{(2-x)(4+x^2+2x)}{(x-2)(x+2)} = \frac{-12}{4} = -3$

ب) $\lim_{x\to \pi} \frac{|\sin x|}{\sin 2x}$

$\pi^+ \to \frac{-\sin x}{\sin 2x} = \frac{-\cancel{\sin x}}{2\cancel{\sin x}\cos x} = \frac{-1}{2\cos x} = \frac{1}{2}$

$\pi^- \to \frac{\sin x}{\sin 2x} = \frac{\cancel{\sin x}}{2\cancel{\sin x}\cos x} = \frac{1}{2\cos x} = -\frac{1}{2}$

۸

الف) $\lim_{x \to \frac{3\pi}{2}} \sqrt{\cos x}$ $\xrightarrow{\frac{3\pi}{2}^+}$ $\sqrt{0^+} = 0$

$\xrightarrow{\frac{3\pi}{2}^-}$ $\sqrt{0^-}$ تعریف نشده

ب) $\lim_{x \to 0} \sqrt{x - \sqrt[3]{x}}$ $\xrightarrow{0^-}$ تعریف نشده

0^- $\Rightarrow x < \sqrt[3]{x}$ $\Rightarrow x - \sqrt[3]{x} < 0$ تعریف نشده

۹

الف) $\lim_{x \to 0^+} \frac{-1}{x^3 - x^4} \overset{[\sin x]}{=} \frac{1}{x^3(x-1)} = \frac{1}{-0^+} = -\infty$

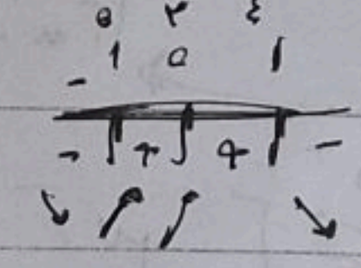
$0^+ - 1 < 0$

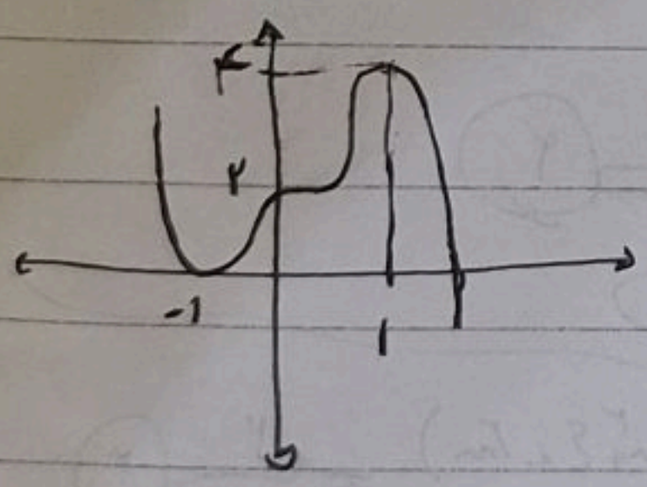
ب) $\lim_{x \to 2} \frac{x^2 - [x^2]}{x - [x]}$ $\xrightarrow{2^+}$ $\frac{x^2 - 4}{x - 2} = x + 2 = 4$

2^- $\frac{4-3}{2-1} = 1$

۱۰

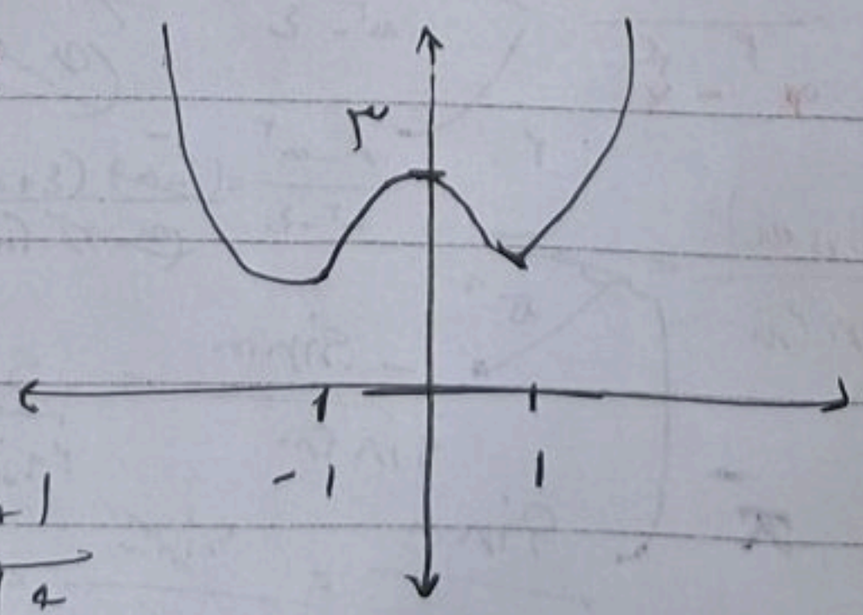
الف) $y = 5x^3 - 3x^5 + 1$ $\xrightarrow{\text{مشتق}}$ $y' = 15x^2 - 15x^4 = 15x^2(1 - x^2)$

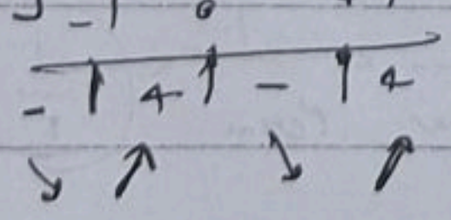




ب) $y = x^4 - 2x^2 + 3$

$4x^3 - 4x = 4x(x^2 - 1)$





Sibó