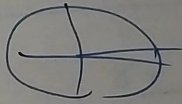


1. $\lim_{x \rightarrow a} b \sqrt{x^2 - 1}$ L'Hôpital's rule

① $x \left[\frac{x-a}{x} \right] + a \left[\frac{x+a}{2} \right]$

$-x^- \rightarrow x \left[\frac{x+x^-}{x} \right] + a \left[\frac{-x^-+x}{2} \right] \rightarrow x + -a$

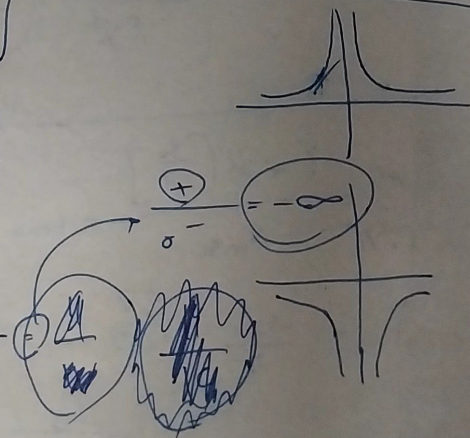
$-x^+ \rightarrow x \left[\frac{x+x^+}{x} \right] + 0 \rightarrow x$ $x + -a = x$
 $a = -x$

② $\lim_{x \rightarrow \frac{\pi}{2}} [x \sin x - 1]$ 
 $\left[1 - 1 \right] = 0$

③ $\lim_{x \rightarrow \frac{1}{e}} [x e^x - x]$ $\left(\frac{1}{e} \right)^e = \frac{1}{e}$ $\left(e^{\frac{1}{e}} - \frac{1}{e} \right) = -1$
 $\left[-1 + \frac{1}{e} \right] = -1$

④ $\lim_{x \rightarrow \left(\frac{1}{e} \right)^-} \log x - 2 + \left[\frac{x}{2x} \right]$
 $\log x - \left[-\frac{1}{2x} \right]$

$\frac{2 - 2 + \left[\frac{1}{2} \right]}{-\frac{1}{2} + 1}$



Q.1

$$\lim_{n \rightarrow \infty} \frac{b \sqrt{r + \sqrt{a}} - rb}{\frac{b}{n} a n - b}$$

Ans

$$\begin{aligned} \text{As } a = b \\ b \frac{\frac{1}{\frac{r + \sqrt{a}}{a}}}{\frac{b}{n} a n - b} &= \frac{1}{\frac{1}{n} - 1} \\ &= \frac{1}{-1} \\ &= -1 \end{aligned}$$

Calculus

2) $\lim_{n \rightarrow 1^+} \frac{\sin^2 \pi n}{[n] + \cos \pi n} = \frac{\sin^2 \pi n}{1 + \cos \pi n} = \frac{1 - \cancel{\cos^2}}{1 + \cancel{\cos}} = 1 - \cos \pi n$
 $\neq 2$

3) $\lim_{n \rightarrow -1} \frac{\sqrt{n+2} - \sqrt{n+1}}{1 + \sqrt{n}} \Rightarrow \frac{\sqrt{n+2} - \sqrt{n+1}}{\frac{1}{\sqrt{n}}} = \frac{-1}{\frac{1}{\sqrt{n}}} = -\sqrt{n}$

4) $\lim_{n \rightarrow 1} \frac{2n - \sqrt{5n} + 8}{2n - \sqrt{5n+1}} = \frac{2 - \sqrt{5}}{2 - \sqrt{6}} = \frac{-\frac{1}{\sqrt{5}}}{\frac{2 - \sqrt{6}}{\sqrt{5}}} = \frac{-1}{2 - \sqrt{6}}$

5) $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{b} + \sqrt{bn + c}}{n} = \frac{1}{\epsilon} \quad \frac{b}{\sqrt[n]{bn+c}} = \frac{1}{\epsilon n}$
 $\frac{-\sqrt[n]{b} \times b}{\epsilon \sqrt[n]{b}} = \left(\frac{-1}{\sqrt[n]{b}} \right)$
 $\sqrt[n]{b} = \sqrt[n]{c}$
 $\sqrt[n]{b} = c$
 $a = -\sqrt[n]{b}$
 $a = -\sqrt[n]{c}$

6) $\lim_{n \rightarrow \infty} \frac{\{x\} + k \left[\frac{x}{\pi} \right]}{\sin x} = +\infty$



$k = 2$

$\frac{-\frac{1}{\pi} \times \frac{1}{\pi} - \frac{1}{\pi} \times \frac{1}{\pi}}{\frac{1}{\pi}} = \left(\frac{-2}{\pi} \right)$

$\lim_{n \rightarrow \infty} \frac{\{x\} + k \left[\frac{x}{\pi} \right]}{\sin x} = +\infty$
 $\frac{1}{\pi} < k < 2$
 $\{x\} = \frac{1}{\pi}$