آیین غضنفری سواد کوهی

ثابت = $y=a$

۱ـ $f(x)=(x+1)^2+(3x-1)^2+mx^2+nx+mn$

اگر تابع ثابت باشد ضرایب x^2 و x باید صفر باشد ← $f(x)=10x^2-4x+2+mx^2+nx+mn$

$\rightarrow m=-10\ ,\ n=4 \rightarrow g(x)=\{(4,-10),(4,p+1),(p+2,-1),(-9,p+k)\}$

$g \rightarrow$ تابع $\rightarrow -10=p+1 \rightarrow p=-11 \quad p+k=-1 \quad k=10$

$-10-11+10+4=-7$

۲ـ الف) $\sqrt{\dfrac{x-2}{x^4-2x^2-8}}$ ← ریشه = ۲ $\qquad \dfrac{x-2}{x^4-2x^2-8}\geq 0$

$x^2=t$

$t^2-2t-8=(t-4)(t+2)$ ← $t=-2$ و $x^2=-2$ ✗ ، $t=4$ و $x^2=4 \rightarrow x=\pm 2$

ریشه‌ها: ۲ و −۲ مضاعف

	-2		2	
$-$	ت	$+$	ت	$-$

$\rightarrow D_f=(-2,2)$

ب) $\dfrac{2x^2+1}{4-2[-x]} \rightarrow 4-2[-x]\neq 0 \rightarrow 2\neq[-x] \rightarrow D_f=\mathbb{R}-(-3,-2]$

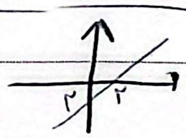
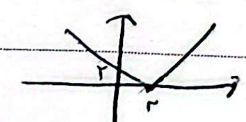
۳ـ الف) $f(x)=\dfrac{\sqrt{x(x^2-1)}}{\sqrt{|x|+x}} \rightarrow x(x^2-1)\geq 0 \rightarrow$

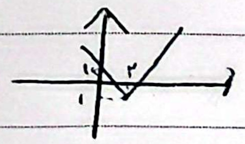
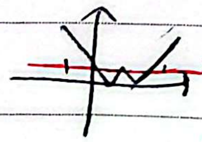
	-1		0		1	
$-$		$+$		$-$		$+$

$[-1,0]\cup[1,+\infty)$ (1)

$\mathbb{R}^+$ فقط (2)

$(1)\cap(2)=D_f=[1,+\infty)$

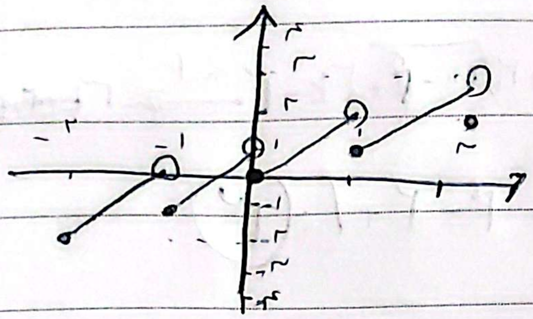
۴ـ $||x-2|-1|\neq k \rightarrow x-2$  $\rightarrow |x-2|$ 

$|x-2|-1 \rightarrow$  $\rightarrow ||x-2|-1|=k$  $\rightarrow k \rightarrow k=1$

۳ محل برخورد

۵-

$y=2x-[x] \xrightarrow{\text{مشتق}} y=2$ ← شیب خطوط



۶-

$f(x)=\dfrac{x^2-4x+3}{x+a}+b$ ← تابع همانی ← $y=x$

$\dfrac{(x-1)(x-3)}{x+a}+b \xrightarrow{a=-1} x-3+b=y \to b=3 \qquad -1+3=2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2-2=0$

$\qquad\qquad\qquad \xrightarrow{a=-3} x-1+b=y \to b=1 \to 1-3=-2$

۷-

~~$f(x)=\begin{cases}\dfrac{x^3+2x^2-x-2}{x^2-1} & ; x\neq\pm1\\ \dfrac{ax+2}{x+b} & ; x=\pm1\end{cases}$~~

~~$\dfrac{x^3+x^2+x+1+x^2-2x-2}{x^2-1}=\dfrac{(x+1)^3+(x+1)(x-3)}{(x+1)(x-1)}=\dfrac{(x+1)(x^2+2x+1+x-3)}{(x+1)(x-1)}$~~

~~$(x+1)(x-1)$~~

$f(x)=\begin{cases}\dfrac{x^3+2x^2-x-2}{x^2-1} & ; x\neq\pm1\\ \dfrac{ax+2}{x+b} & ; x=\pm1\end{cases}$ → $x^3+2x^2-x-2 \,\big|\, x-1$ → $x^2+3x+2=(x+1)(x+2)$

$\dfrac{(x+1)(x+2)(x-1)}{(x+1)(x+2)}=x+2$

$\underline{f=g}$

$\underline{c=2} \qquad g(1)=3 \qquad \dfrac{a+2}{1+b}=3 \to a+2=3+3b$

$\qquad\qquad g(-1)=+1 \qquad\qquad\qquad a-3b=1 \qquad$ جمع کردیم $\dfrac{-2}{-4}=\dfrac{1}{2}=\underline{b}$

$\dfrac{-a+2}{-1+b}=1 \to \begin{array}{l}+a+b=+3\\ -a+2=-1+b\end{array} \qquad \underline{c+b=2/5}$

۸- $2f-g=\{(1,k)\} \to D_{f-g}=\{1\}$ ، $D_f=[\frac{۳}{۴}a,+\infty)$ $D_g=(-\infty,\frac{b}{۲}]$

$1=\frac{b}{۲}=\frac{۳}{۴}a \to \underline{b=۲}$, $\underline{a=\frac{۴}{۳}} \to ۲f-g=\sqrt{۴-۴x}-۳k+۲\sqrt{۴x-\frac{۶}{۴}}+۳k-۲ \xrightarrow{x=1} -۳ \cancel{k}$

$۲k-۲-۳k=k \to \underline{k=-1}$ $\quad ۳a-\frac{b}{۲}-k=۴-۲+1=۳$

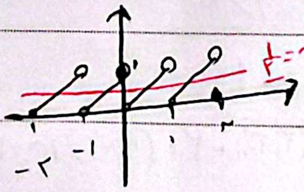
۹- $f\times g=(\sqrt{۲x+۲})(\sqrt{m-x}+x) \to ۲x+۲\geq ۰ \to x\geq ۱$

$m\geq x$ $\quad D_f=[۱,m] \to m=۷$

$f(۲)\cdot g(۲)=۶\sqrt{۲}:\sqrt{۶}+n-\sqrt{۸}=۲+n-۲\sqrt{۲}=۶\sqrt{۲} \to n=۸\sqrt{۲}-۲$

$m+n=۸\sqrt{۲}-۲+۷=۵+۸\sqrt{۲}$

۱۰- $y=(-1)^{۲}(x-[x])=\frac{۱}{۲} \to (x-[x])=\frac{۱}{۲}$



۲ جواب ←